THE OPERATION OF ALLOY JUNCTION TRANSISTORS AS LARGE SIGNAL PULSE AMPLIFIERS IN THE ACTIVE REGION

Emera Sherburne Bailey

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Emera S. Bailey

Letter on cover:

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by

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United States Naval Postgraduate School

PREFACE

The study from which this paper emerged was performed in early 1956 during an industrial tour with the Radio Corporation of America, Semi Conductor Division, currently located at Harrison, New Jersey.

It represents the beginnings of a continuing program by this division to determine the optimum parameters for operation of transistors in pulse applications, for example, their use in computers.

This phase was considered concluded with the evaluation of a method of attack described herein which permitted isolation of the significant parameters for the grounded emitter stage. This may easily be extended to include the grounded base and grounded collector configurations. A similar approach suggests itself for application to the saturated regions as well.

The writer wishes to thank Mr. C. Frank Wheatly for suggesting the theoretical approach used herein which he had successfully employed in noise studies. He also wishes to thank Mr. A. Lyle Cleland for his efforts in setting up the program, his guidance throughout, and his careful evaluation of the results.

He also wishes to thank Dr. L. J. Giacoletto for his assistance in the derivation of the theoretical relations contained in Appendix 2.

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TABLE OF SYMBOLS

A'	Intrinsic Voltage Gain
$C_{b'c}$	Collector capacity to the Intrinsic Base
$C_{b'e}$	Diffusion Capacity to the Intrinsic Base
C_c	Collector capacity to Intrinsic Base
C_{gk}	Grid to Cathode Capacity
C_{gp}	Grid to Plate Capacity
C_{T_c}	Collector Transition Capacity
C_{T_e}	Emitter Transition Capacity
D_p	Diffusion Constant for p-type Germanium
E_{cc}	Collector Supply Voltage
$E(t)$	Open circuit Applied Step of Voltage
$g_{b'c}$	Collector Leakage Conductance to the Intrinsic Base
$g_{b'e}$	Diffusion Conductance to the Intrinsic Base
g_{ce}	Collector to Emitter Conductance
g_m	Base to Collector Transconductance
I_b	Final Value of Base Current
I_B	DC Value of Base Current
I_c	Final Value of Collector Current after the Step or Before the Step but not Including the Quiescent Current
$\bar{I}_c$	Total Collector Current
I_C	DC Collector Current
I_{co}	Collector Saturation Current When Both Junctions are Biased in the Reverse Direction
I_{c2}	Arbitrary Current Assumed in Second Approximation for Rise Time in Chapter IV

$I_{c'}$	Total Collector Current
$I_{c'c'}$	Output Conductance Coefficient Divided by
$I_{c'e'}$	Forward Transfer Conductance Coefficient Divided by
$I_{c's}$	Same as I_{co} approximately
I_E	DC Emitter Current
eI_o	Current that Flows in the Collector to Emitter Loop With the Base to Emitter Voltage Equal to $\frac{1}{\lambda}$ Forward Bias and Normal Collector to Base Reverse Bias Applied Greater than .5 Volt.
I_3, I_4	Collector Quiescent Current. I_3 used in Rise Time Work, I_4 in Fall Time
k	Boltzmann's Constant
K_e	Relative Permitivity
L_b	Diffusion Length of the Minority Carrier in the Base Region
q	Charge of the Minority Carrier
$r_{bb'}$	Base Lead Resistance or Base Spreading Resistance
R_g	Generator Resistance
R_L	Load Resistance (Collector)
r_p	Dynamic Plate Resistance
T	Absolute Temperature
T	Arbitrary Time Constant
T_F	Fall Time
T_R	Rise Time
t_1	Time at the 10% or 90% Point (Rise or Fall Respectively)
t_2	Time at the 90% or 10% Point (Rise or Fall Respectively)
$V_{b'e}$	Voltage from Intrinsic Base to Emitter
$V_{c'b'}$	Voltage from Collector to Intrinsic Base
V_{ce}	Voltage from Collector to Emitter

V_{ceo}	Collector to Emitter Voltage Where C_c Measured
$V_{e'b'}$	Voltage from Emitter to Intrinsic Base
V_{gk}	Voltage from Grid to Cathode
W_b	Base Width between Points of Zero Electrostatic Potential Gradient
α_{cb}	Collector to Base Current Gain as a Number Greater than 1 at 1 KC
$\alpha_{cb}(\omega)$	Collector to Base Current Gain as a Function of Frequency
α_{ce}	Collector to Emitter Current Gain at 1 KC
$\alpha_{ce}(\omega)$	Collector to Emitter Current Gain as Function of Frequency
ΔE	Step of Voltage
ΔI	Step of Current
ε_o	Permitivity of Free Space
Λ	q/kT
μ_n	Mobility of Electrons
σ_b	Conductivity of Base Material
ω_{cb}	Radian Frequency Where α_{cb} Magnitude is .707 of the Value at 1 KC
ω_{ce}	Radian Frequency Where α_{ce} Magnitude is .707 of the Value at 1 KC

CHAPTER I

INTRODUCTION

1. Summary

Existing methods of attacking the problem of transient response of junction transistors in the active region are reviewed and expanded, using the hybrid-pi equivalent circuit. Some weaknesses of too much confidence or reliance on the use of equivalent circuits in any form are pointed out. Parameter variation with operating point is discussed for the parameters of importance. Finally, a new approach to the problem is suggested that is based on the use of equivalent circuits, but includes relationships to account for the variation of some parameters with operating point and holds for large signals and most circuit configurations of practical importance.

2. Definitions

The active region of operation of junction transistors is best defined using the work of W. M. Webster, M. C. Kidd and W. Hasenberg "Delayed Collector Conduction, A New Effect in Junction Transistors".(1) Their drawing of the regions of operation using the collector output characteristics is reproduced as Figure 1.

The region under consideration in this paper and called the active region is essentially regions II and III. Careful attention was paid to not permitting operation in other regions, especially Region I, the saturation region.

The IRE Standards on Pulses: Definitions of Terms, 1951(4) defines:

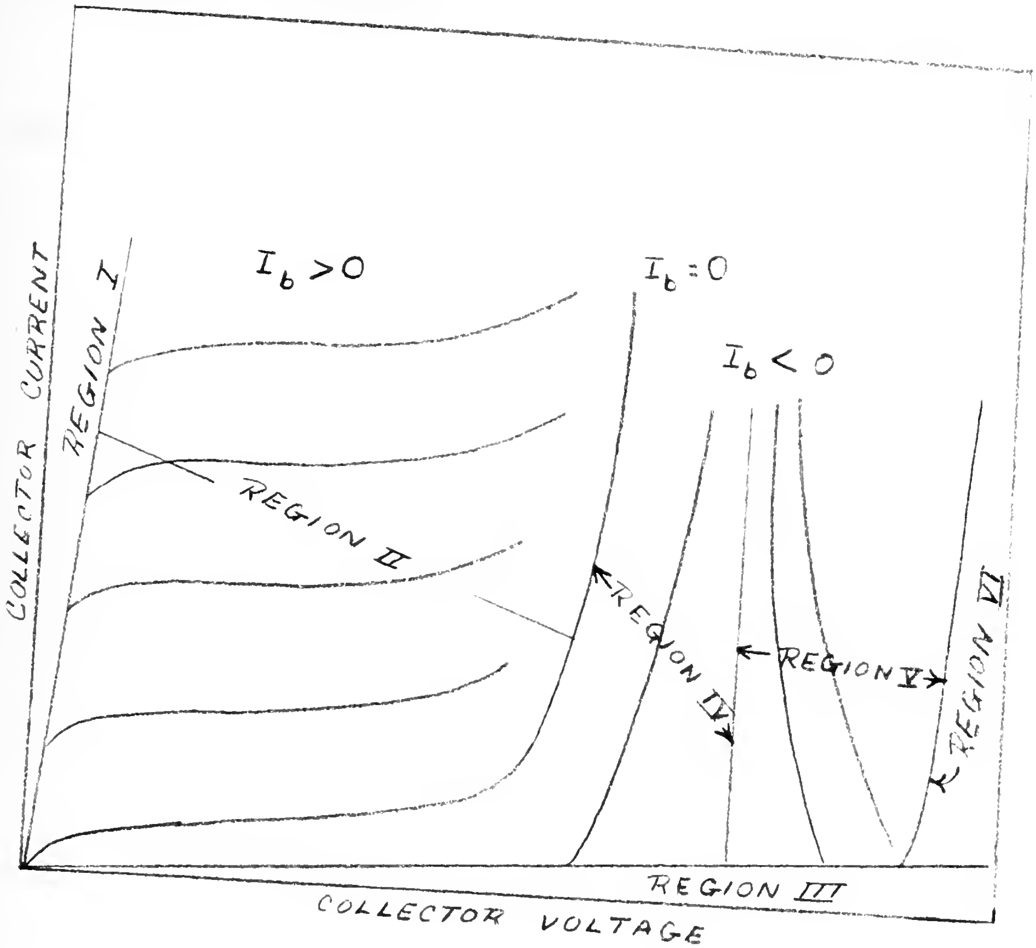


Figure 1

Region I - Saturation

Region II - Constant Current

Region III - Cutoff

Region IV - Constant Voltage

Region V - Negative Resistance

Region VI - Breakdown

Pulse Rise Time - The interval between the instants at which the instantaneous amplitude first reaches specified lower and upper limits, namely, ten per cent and ninety per cent of the peak pulse amplitude unless otherwise stated. Designated T_R in this paper.

Pulse Decay Time - The interval between the instants at which the instantaneous amplitude last reaches specified upper and lower limits, namely, ninety per cent and ten per cent of the peak pulse amplitude unless otherwise stated. Designated T_F in this paper.

The ten per cent to ninety per cent points are used throughout this paper.

3. Background

The subject of pulse amplifiers or video amplifiers using the vacuum tube has received much attention in recent years especially since the advent of television and radar on a large scale.

On the other hand the subject of the transistor as a pulse amplifier has in general been treated only where it is used for computer applications in a saturating condition, either saturation of the transistor or saturation of diodes used in conjunction with the transistor. This is understandable since there have not been suitable units cheaply produceable that could by themselves act as fast switching devices without the use of saturation to improve their rise and fall times. Today the frequency range of transistors is constantly being pushed up until it seems probable that with the advent of the Drift Transistor and other really high frequency devices, the transistor may be expected to come into its

own as a pulse amplifier at reasonable prices. The work of John L. Moll, "Large Signal Transient Response of Junction Transistors"[2], considers the short circuit case for small signal operation in the active region and does so only for the ideal current generator and voltage generator. By using the short circuit case, the author avoids the consideration of the effects of collector capacity except where it limits the short circuit case. He uses the T equivalent circuit for his discussion throughout.

This paper utilizes the hybrid - pi equivalent circuit developed by L. J. Giacoletto, "Study of P-N-P Alloy Junction Transistor from D-C through medium frequencies."[3]

The general procedure followed is to break the study into four parts:

a. Proceding in the manner suggested by Moll and extending the method to include collector capacity and generator resistance.

b. Considerations of the variations of the parameters with operation point.

c. Development of an approach to eliminate some of the variation of parameters with operating point.

d. Comparison of actual laboratory measurements with theory.

4. Resumé

In general the results agreed with theory very well for the proposed approach. The quantities required are easily measured and show that for the range of operation considered, the essential transistor parameters required to define the unit completely are α_{cb}, ω_{ce} or ω_{cb}, $r_{bb'}$, $C_{b'c}$, I_{co} and I_o (a DC quantity to be defined later). The necessary quantities

from the circuit standpoint are the final value of I_c or I_b, open circuit applied voltage or current amplitude, R_g, R_L and quiescent collector current. When these quantities are known, total I_b, $\bar{I}_c$, T_R, T_F, Voltage gain, etc., may be easily calculated.

CHAPTER II

CALCULATIONS USING SIMPLE SMALL SIGNAL THEORY

1. The Hybrid - pi Equivalent Circuit

The Hybrid - pi equivalent circuit was suggested by L. J. Giacoletto[3] and is reproduced below as Figure 2.

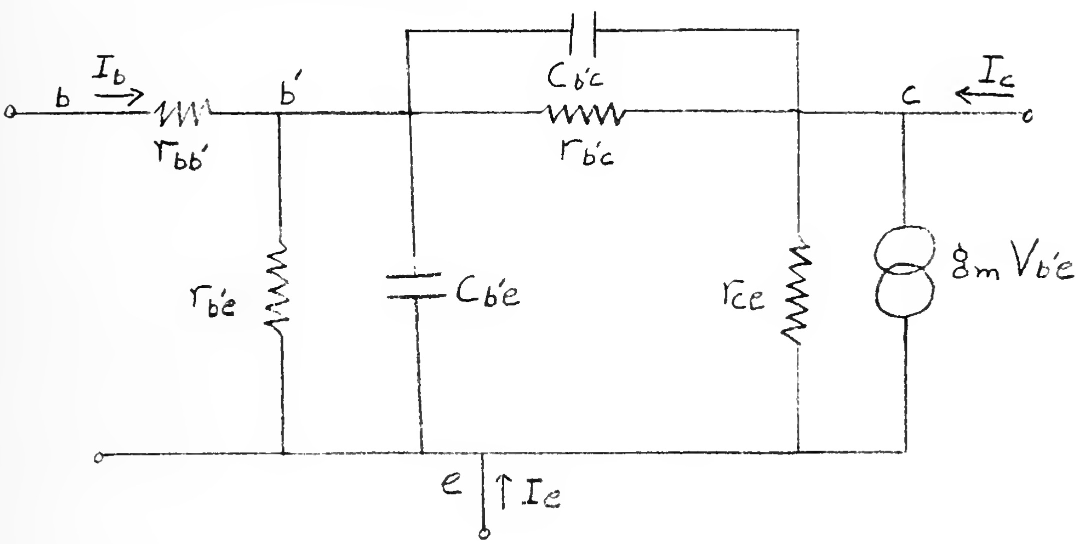


Figure 2

Where:

$r_{bb'}$ = base lead resistance

$$r_{b'e} = \frac{1}{\Lambda I_B}$$

$$C_{b'e} = \Lambda I_E \frac{W_b^2}{2 D_p}$$

$$C_{b'c} = \frac{K_e \varepsilon_o A_c}{\left[\frac{2 K_e \varepsilon_o \mu n V_{ce}}{\sigma b}\right]^{\frac{1}{2}}} - I_c \frac{W_b}{2 D_p} \frac{\delta W_b}{\delta V_{cB'}}$$

$$r_{b'c} = \frac{1}{I_c \frac{W_b}{2L_b^2} \frac{\delta W_b}{\delta V_{cB'}}}$$

$$r_{ce} \doteq \frac{1}{\frac{I_c}{W_b} \frac{\delta W_b}{\delta V_c}}$$

$$g_m \doteq \Lambda I_c$$

$$\frac{\delta W_b}{\delta V_c} \doteq \frac{1}{2} \left[\frac{2 K_e \varepsilon_o \mu_n}{\sigma_b V_{ce}} \right]^{\frac{1}{2}}$$

W_b = base width between points of zero electrostatic potential gradient

L_b = diffusion length of the minority carriers in the base region

D_p = diffusion constant for P-type germanium

Λ = q/kT 38.6 volts^{-1} at 27 degrees centigrade

q = charge of the minority carrier

k = Boltzmann's constant

T = absolute temperature

K_e = relative permittivity (16 for germanium)

ε_o = permittivity of free space

A_c = effective collector area

μ_n = mobility of electrons

σ_b = conductivity of base material

Major assumptions and limitations:

a. $\Lambda V_{ce} > 1$

b. $W_b/L_b \ll 1$, $\left| \frac{1}{\Lambda W_b} \frac{\partial W_b}{\partial V_{EB}'} \right| \ll 1$

c. Limited to the frequency range, $f \ll \frac{6 D_p}{2 \pi W_b^2}$

d. $C_{Te} \ll$ the diffusion capacitance,

where: C_{Te} = the transition capacitance of the emitter junction

e. That the parameters of the equivalent circuit are essentially constant with operating point

2. Analogy with the Vacuum Tube

The easily drawn analogy with the vacuum tube equivalent circuit should not be overlooked, wherein the following table of analogies is pointed out as an aide in considering what is to follow.

TRANSISTOR	VACUUM TUBE
$r_{b'e}$	Transit time resistance
$C_{b'e}$	C_{gk}
$C_{b'c}$	C_{gp}
r_{ce}	r_p
$g_m V_{b'e}$	$g_m V_{gk}$

Typical values for units used:

$r_{b'e} \sim 100$ at $I_c = 10$ ma, 1000 at $I_c = 1$ ma.

$C_{b'e} \sim 1000$ uuf at $I_c = 1$ ma.

$C_{b'c} \sim 10$ uuf at $V_{ce} = 6$ volts

$g_m \sim .04$ at 1 ma. , .4 at 10 ma.

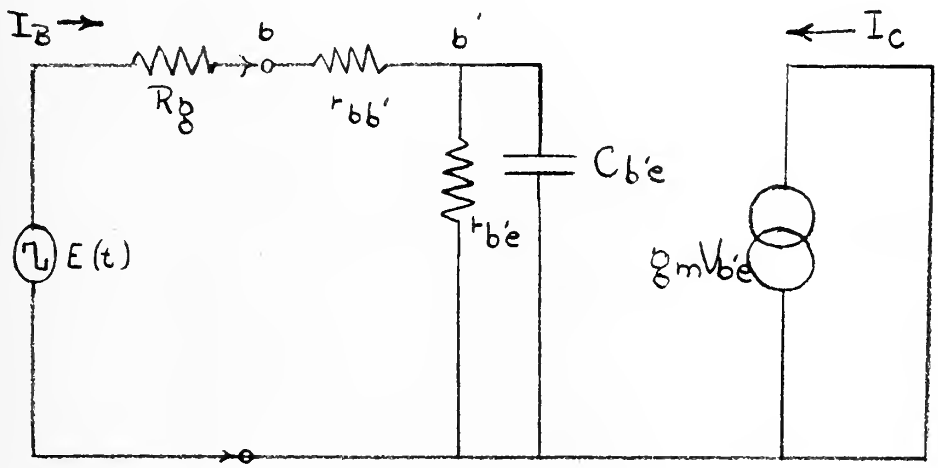
$r_{ce} \sim .5$ megohm or greater

$r_{b'c} \sim$ megohms

3. The Short Circuit Case

Using the grounded emitter circuit, the simplest possible approach

is to assume that the load resistance, R_L, is so small that it appears like a short circuit. Then the circuit of Figure 3 remains:



ASSUMES: $C_{b'c} << C_{b'e}$

$g_m R_L C_{b'c} << C_{b'e}$

Figure 3

If E(t), the open circuit voltage, is assumed a step, the following results may be derived (see Appendix 1 for the derivation) (It should be noted that a step of current with associated shunt conductance will give similar results since the step is assumed under open circuit conditions):

$$I_c = \frac{\alpha_{cb}\, E}{r_{bb'} + R_g + r_{b'e}} \left[1 - e^{-\left(\frac{r_{b'e}}{r_{bb'} + R_g} + 1\right)\omega_{cb} t} \right]$$

$$T_R = T_F = \frac{2.2}{\omega_{cb}} \left[\frac{1}{1 + \frac{r_{b'e}}{r_{bb'} + R_g}} \right]$$

Where: ω_{cb} = the radian cutoff frequency where α_{cb} is .707 times the low frequency value

α_{cb} = short circuit current gain of the grounded emitter stage.

This result agrees with the derivations of J. L. Moll[2] since in the limiting cases of an ideal current generator, R_g equals infinity and $T_R = T_F = 2.2/\omega_{cb}$ which agrees generally with Moll's derivation for a step of current ΔI. In the other case of an ideal Voltage generator, $R_g = 0$, and:

$$T_R = T_F = \frac{2.2}{\omega_{cb}} \left[\frac{1}{1 + r_{b'e}/r_{bb}} \right]$$

This is also in agreement with Molls derivation for a step of voltage ΔE. Both of these agree exactly with Moll's theory provided the assumption $\omega_{cb} \doteq \frac{\omega_{ce}}{\alpha_{cb}}$ is made.

4. Intermediate Case

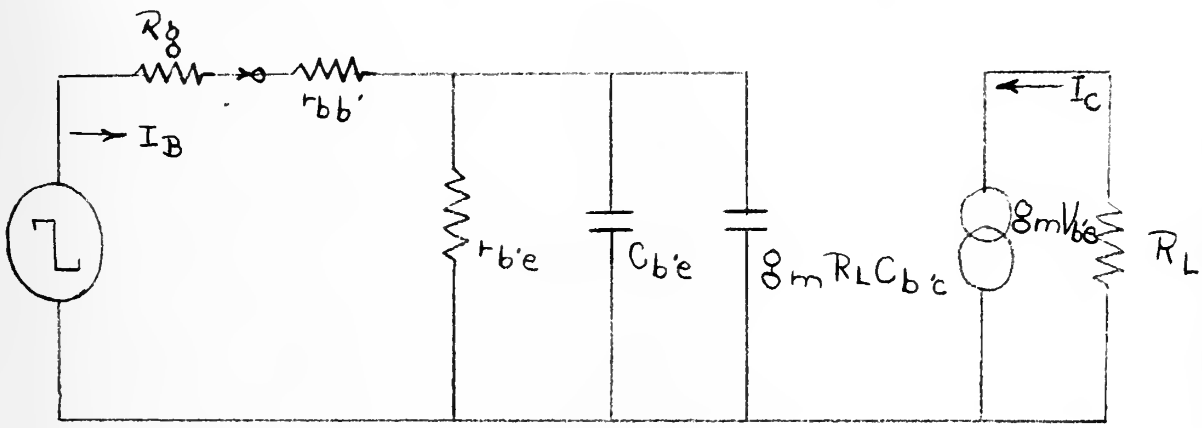
The next approximation to the short circuit case is suggested by the similarity between the feedback capacitor $C_{b'c}$ and C_{gp} of the vacuum tube. Due to the "Miller Effect" described in "Principles of Radar"[5] pages 229 and 230, the capacitor $C_{b'c}$ is effectively a larger one tied between b' and the emitter equal to $C_{b'c}$ multiplied by the intrinsic voltage gain of the stage plus one. Since:

$$A' = g_m R_L \quad \text{and} \quad g_m R_L >> 1$$

for all cases in question, effective capacity between intrinsic base and emitter becomes:

$$g_m R_L C_{b'c}$$

This gives the simplified equivalent circuit of Figure 4.



$$\text{ASSUMES: } \frac{r_{b'e}}{R_g + r_{bb'}} \left(C_{b'e} + g_m R_L C_{b'c} \right) >> C_{b'c} R_L$$

Figure 4

Again applying a step of voltage or current as before yields (See Appendix 1):

$$I_c = \frac{\alpha_{cb} E}{r_{bb'} + r_{b'e} + R_g} \left[1 - e^{-\left(\frac{\frac{r_{b'e}}{r_{bb'} + R_g} + 1}{1/\omega_{cb} + \alpha_{cb} R_L C_{b'c}} \right) t} \right]$$

$$T_R = T_F = 2.2 \left[\frac{1}{1 + \frac{r_{b'e}}{r_{bb'} + R_g}} \right] \left[\frac{1}{\omega_{cb}} + \alpha_{cb} R_L C_{b'c} \right]$$

These results agree with Moll's definition of the boundary of the region for the short circuit case. For the units used in comparison with theory, the $C_{b'c}$ term had to be considered from about $R_L = 200$ ohms for the accuracies desired.

5. Final Case

From a consideration of the intermediate case it can be seen that the other extreme of the short circuit case is when R_L becomes very large. Hence, if:

$$\alpha_{cb} R_L C_{b'c} >> 1/\omega_{cb}$$

Then $C_{b'c}$ and R_L will determine the rise and fall times of the stage. For the units used in this study, at $R_L = 5000$ to 10000 ohms, the factor $\alpha_{cb} R_L C_{b'c}$ becomes most significant.

6. Conclusions and Comments on this Approach

The major problems and other significant comments on this approach are as follows:

a. The T_F and T_R are equal only at one particular bias level and this is different for different bias levels of different units. This is especially true at zero or small reverse bias on the emitter - base junction when T_F is much less than T_R.

b. Where R_g is very small, a value must be determined for $r_{b'e}$ and this value is a function of emitter current. This variation is discussed at length in Chapter 3.

c. This approach assumes a linear relationship between I_c and $V_{b'e}$,

but actually g_m itself is a function of current:

$$g_m \doteq \Lambda I_c$$

The more accurate expression here is:

$$I_c \doteq I_o e^{V_{b'e}}$$

The use of the first approximation here can lead to appreciable errors at large signals.

d. When Moll presented his work, he intended to use the results for operation into the saturated region and hence the length of time actually in the active region was very short. This leads to smaller errors in the results. Where large signals are considered wholly in the active region, these troubles are greatly magnified.

e. Variations of other parameters are important in large signal operation and are considered in detail in the next chapter.

f. If it becomes possible to make transistors where diffusion capacity is small and the transition capacity C_{Te} becomes significant, the $C_{b'c}$ term, derived from the other transition capacitance C_{Tc}, will become the more significant term at even low values of R_L.

CHAPTER III

VARIATION OF PARAMETERS WITH OPERATING POINT

1. Theory of Interest

The parameters concerned with the previously derived relations are herein listed with their approximate equivalence to the physical transistor. The expressions for α_{cb}, α_{ce}, ω_{cb}, and ω_{ce} are derived expressions, the derivations contained in Appendix 2. The remainder are merely the expressions listed in Chapter II with the essential operating point variables the only quantities of interest at this point, and hence all other symbols in the original formulas are replaced with a simple K in each case. The figures given parenthetically after each equation are plots of the dependent variable such as $C_{b'e}$ as the ordinate, and the independent variable associated therewith, in this case I_e, as the abscissa.

$$g_{b'e} = 1/r_{b'e} \doteq K\, I_B \quad \text{(Figure 5.)}$$

$$C_{b'e} \doteq KI_E \quad \text{(Figure 6.)}$$

$$C_{b'c} \doteq K/\sqrt{V_{ce}} \quad \text{(Figure 7.)}$$

$$g_m \doteq KI_c \quad \text{(Figure 12.)}$$

$$\alpha_{ce} \doteq -\frac{g_m + g_{ce}}{g_{b'e} + g_m + g_{ce}} \quad \text{(Figure 8.)}$$

$$\alpha_{cb} \doteq -\frac{g_m - g_{b'c}}{g_{b'e} + g_{b'c}} \quad \text{(Figure 9.)}$$

$$\omega_{ce} = -\frac{g_{b'e} + g_m}{C_{b'e}} \qquad \text{(Figure 10.)}$$

$$\omega_{cb} \doteq g_{b'e}/C_{b'e} \qquad \text{(Figure 11.)}$$

$$g_{b'c} \doteq I_c K/\sqrt{V_{ce}}$$

$$g_{ce} \doteq I_c K/\sqrt{V_{ce}}$$

Where the K as used in the above expressions is different in each case.

2. Curves of Measured Parameter Variations with Operating Point (Figures 5 to 12)

Only significant parameter variations are plotted. Little variation of $r_{bb'}$ with operating point is noted except a sharp increase at very low I_c. This is not considered significant to this discussion, but should be considered if exceptionally small currents are involved, that is, currents less than 1 ma. total.

The variations of g_{ce}, and $g_{b'c}$ were not plotted since they are only of interest here to the extent that they influence α_{cb} and α_{ce} which were plotted.

3. Correlation with Theory

No attempt is made to correlate the data that follows with theory. This is adequately covered in the references. These curves are for alloy junction transistors and give a good indication of the variations to be expected for units of this type.

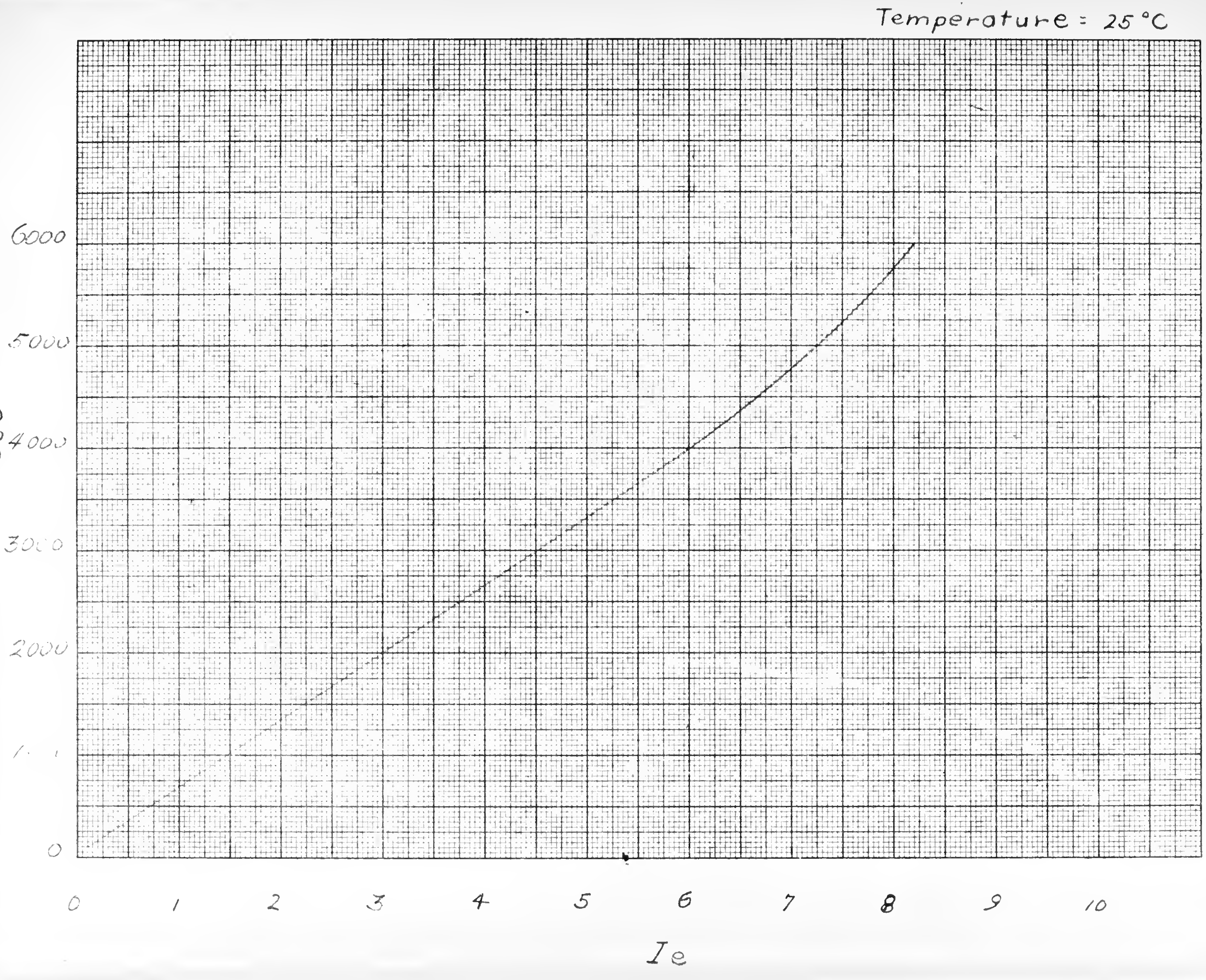

Temperature = 25 °C
$g_{b'e}$ micromhos
6000
5000
4000
3000
2000
0
0
1
2
3
4
5
6
7
8
9
10
I_e

Figure 5

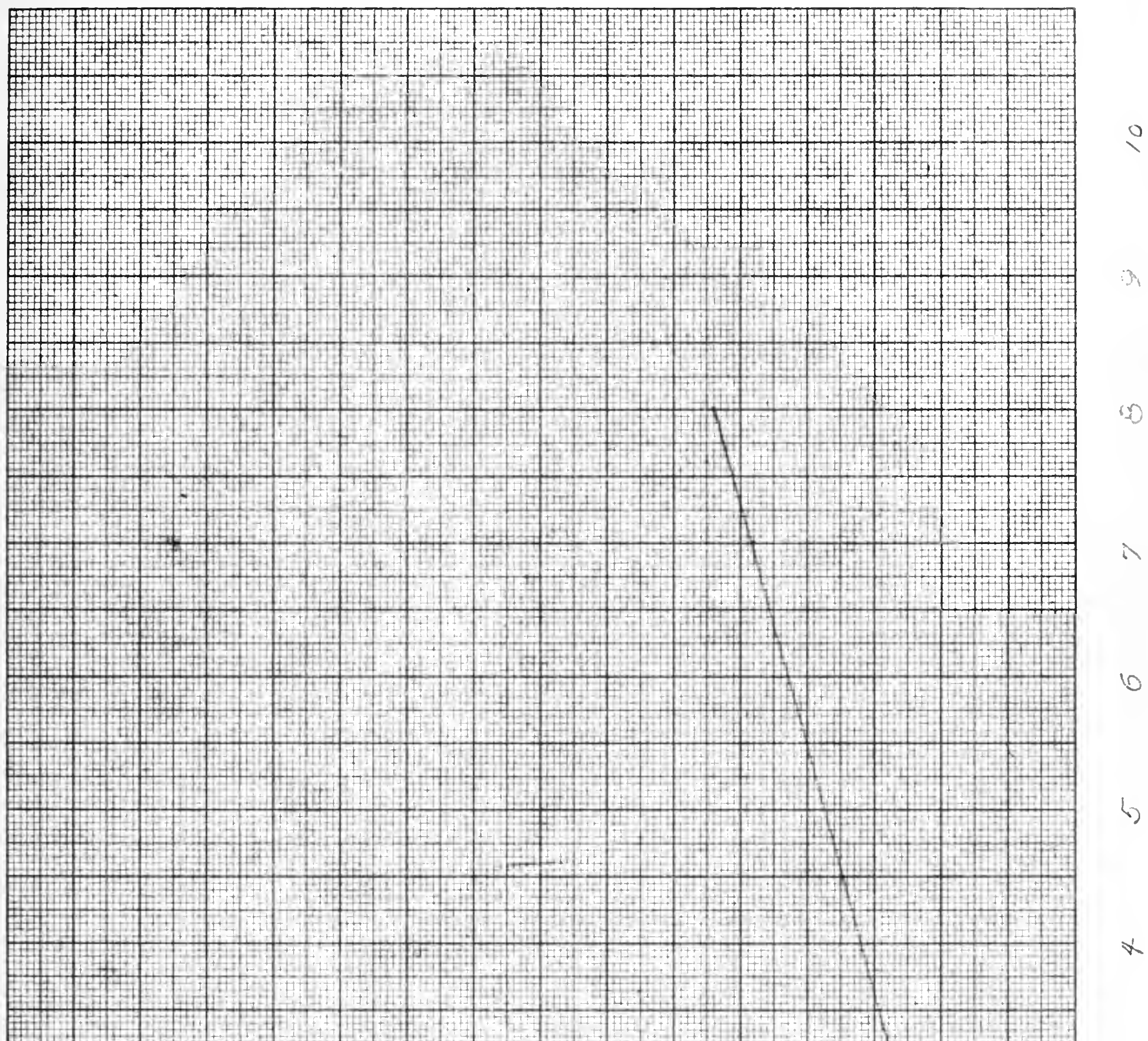
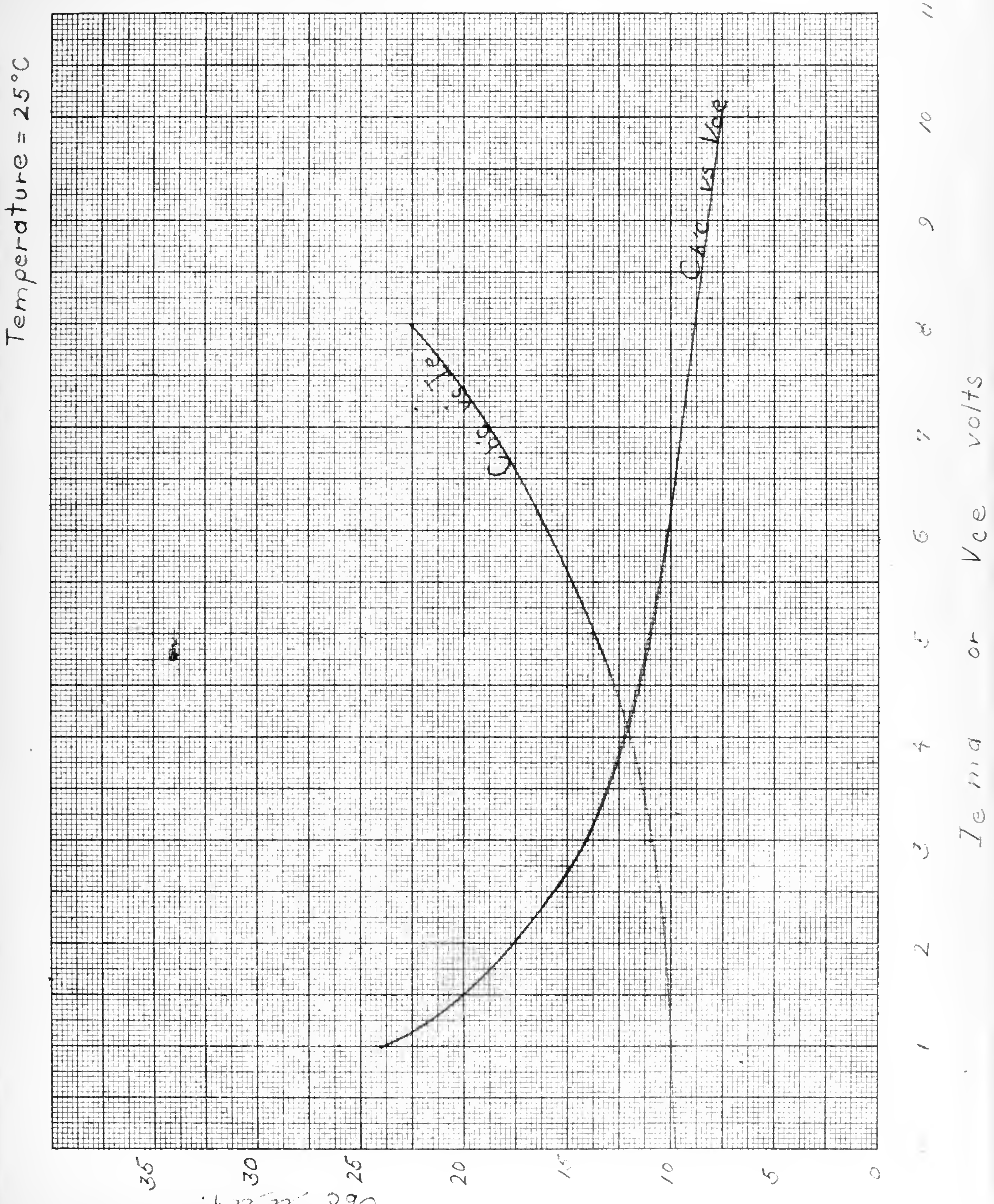
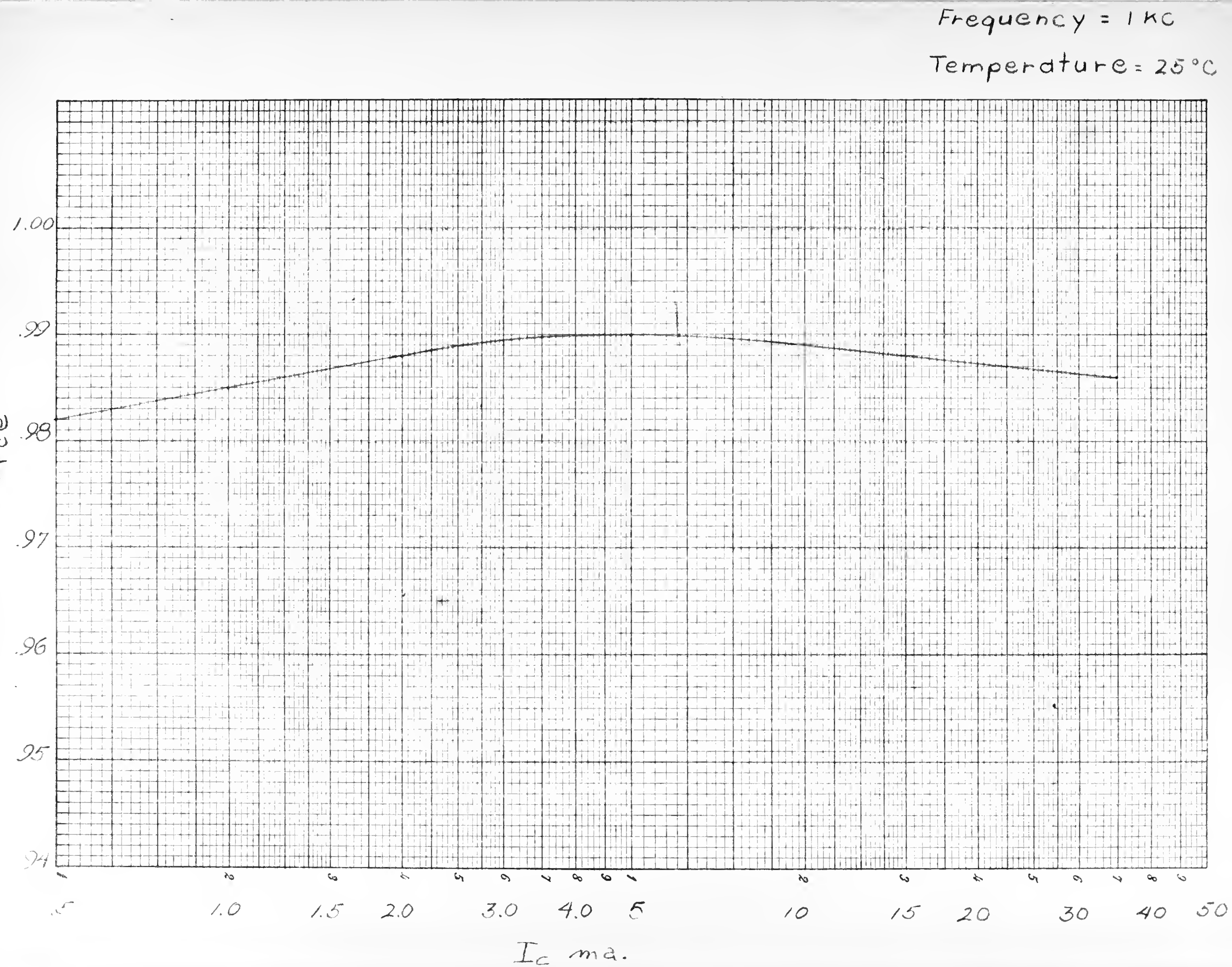


Figure 6.



Temperature = 25°C
Cbc vs Vce
Cbc vs Ie
Ie ma or Vce volts
Cbc μμf.
1 2 3 4 5 6 7 8 9 10 11
35 30 25 20 15 10 5 0

Figure 7.



Frequency = 1 KC
Temperature = 25 °C
1.00
.99
.98
.97
.96
.95
.94
r_{ce}
.5
1.0
1.5
2.0
3.0
4.0
5
10
15
20
30
40
50
I_c ma.

Figure 8.

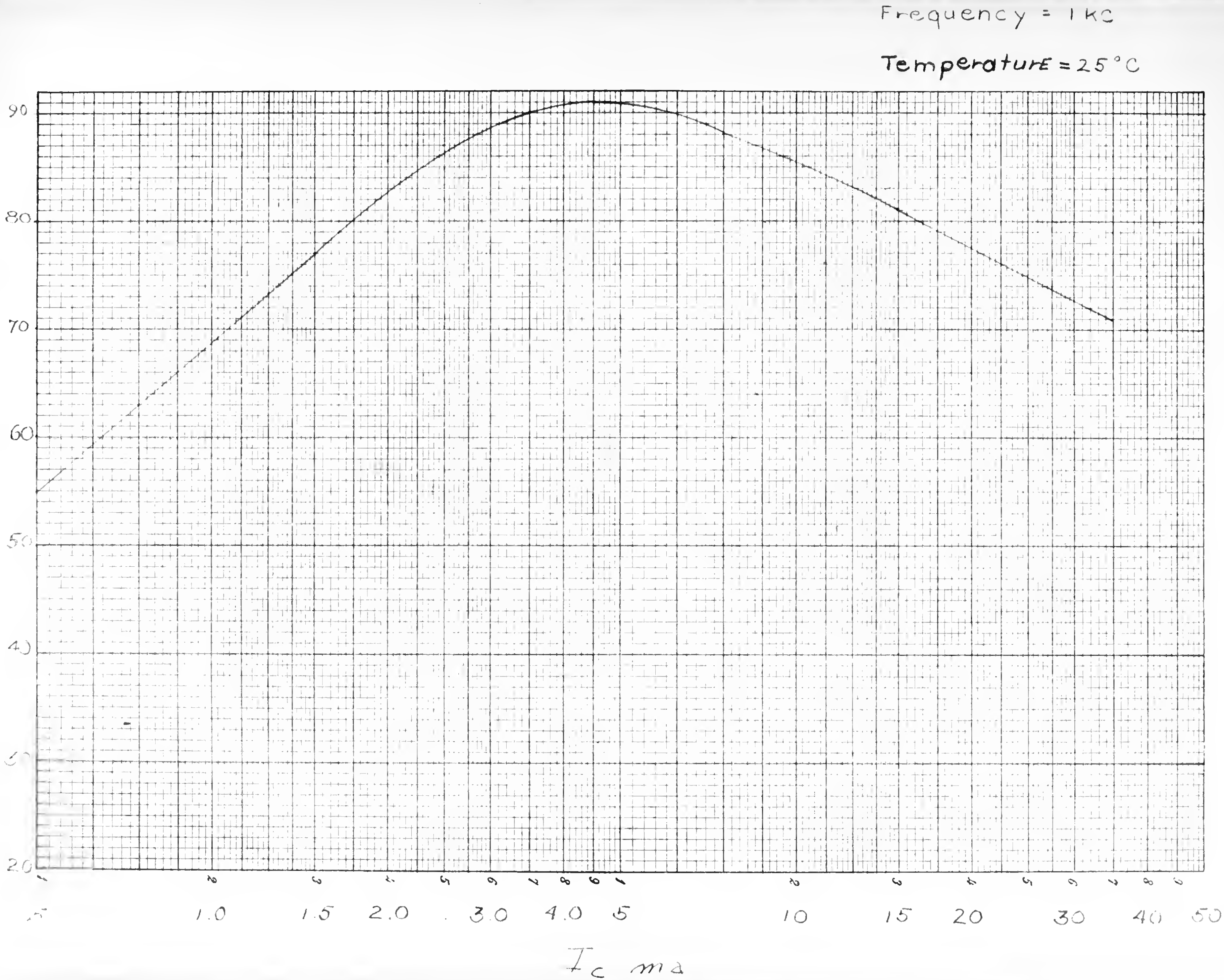

Frequency = 1 KC
Temperature = 25 °C
90
80
70
60
50
40
20
1.0
1.5
2.0
3.0
4.0
5
10
15
20
30
40
50
I_c ma

Figure 9.

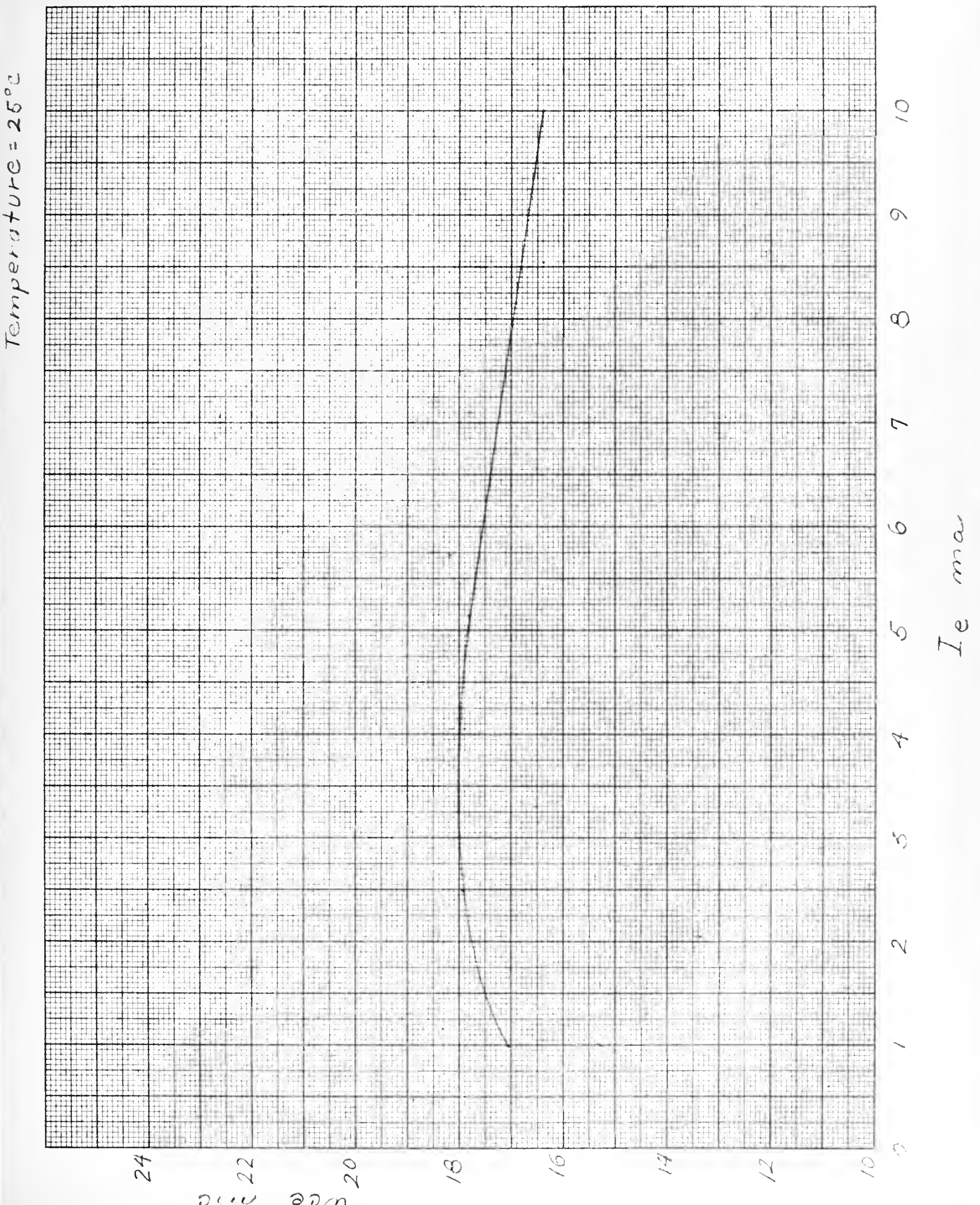


Figure 10

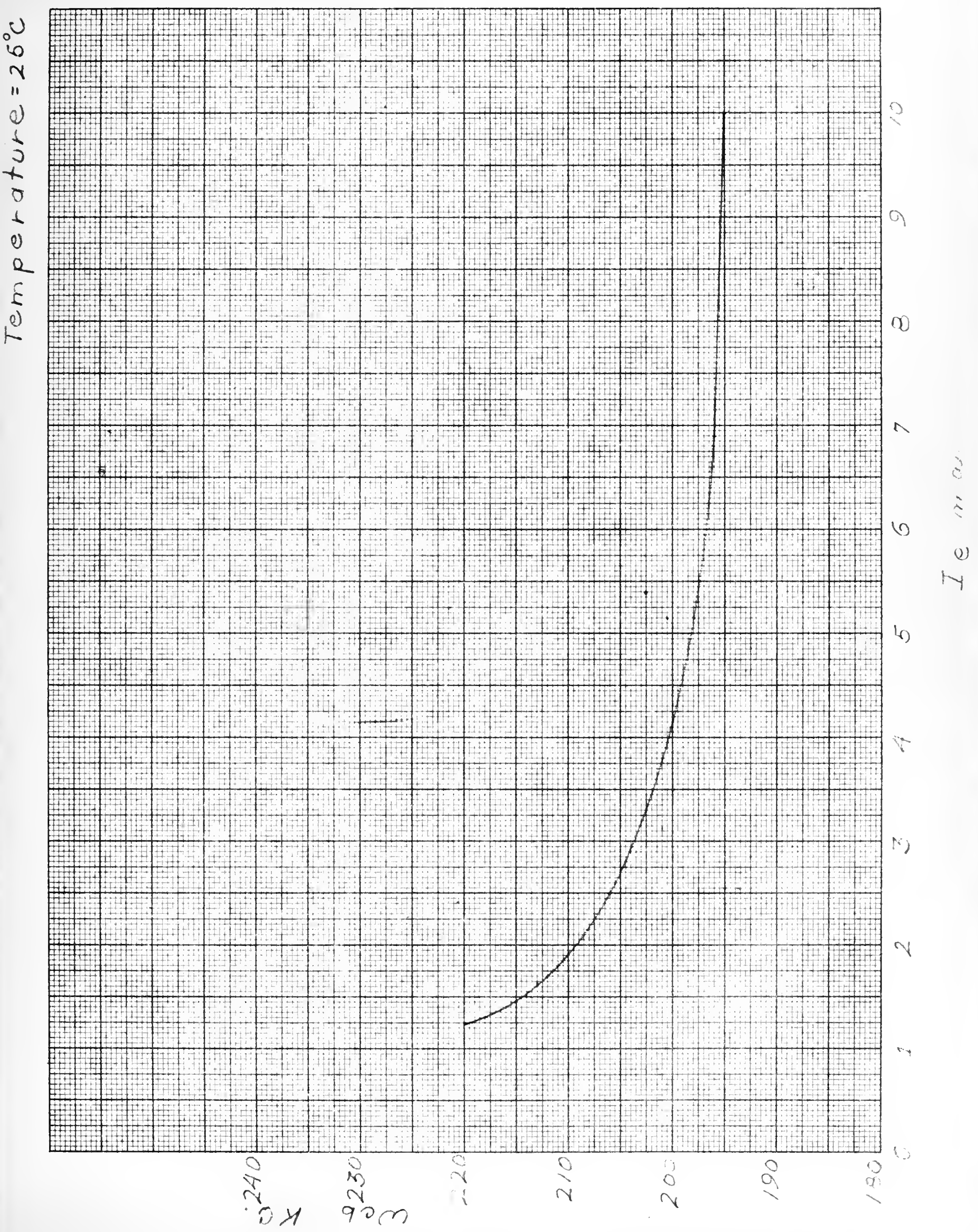

Temperature = 25°C
Ie ma
1
2
3
4
5
6
7
8
9
10
240
230
220
210
200
190
180

Figure 11.

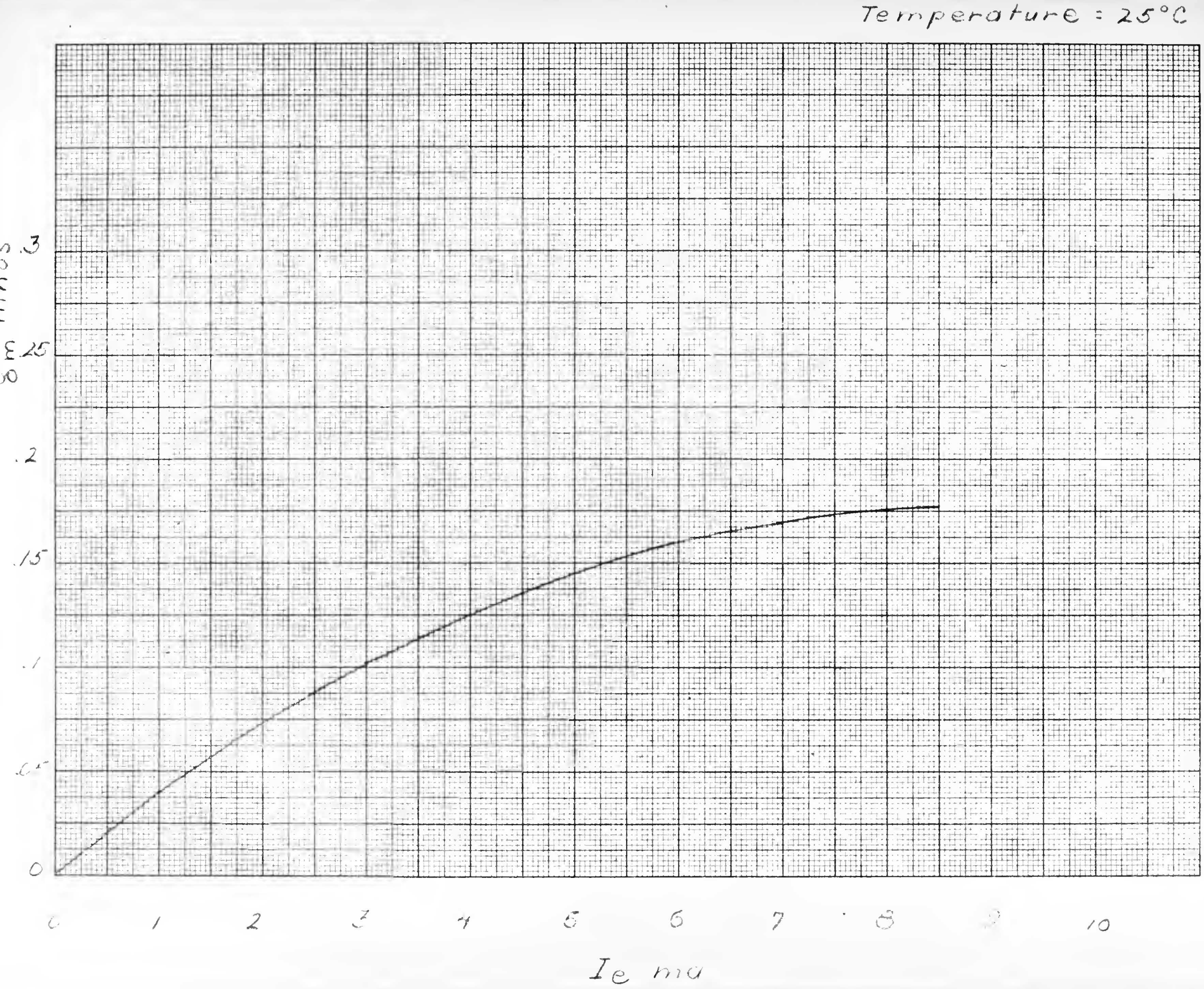


Figure 12.

4. Methods for Evaluating Parameters that Vary with Operating Point in Large Signal Operation

The following methods were evolved for treating those parameters that vary with operating point:

$r_{b'e}$ --- The average value and time mean were both tried with the latter yielding the best results. Values were calculated using the relationship that $r_{b'e} = i_b/\Lambda$ where i_b is the instantaneous value of base current. These methods were especially unsuccessful for fall time and for rise time where $R_g < 500$ ohms.

$C_{b'e}$ --- Not needed

$C_{b'c}$ --- The value of C_c measured with the emitter - base loop open in the grounded base configuration is very nearly equal to $C_{b'c}$ at the measurement voltage, V_{ceo}. Other values of $C_{b'c}$ may be obtained from the relationship:

$$C_{b'c} \doteq C_c \sqrt{\frac{V_{ceo}}{V_{ce}}}$$

The value for the average V_{ce} during the step was found to give satisfactory results for rise time and 1.5 times this value for fall time. For the situation where the $C_{b'c}$ term becomes the controlling factor, the time mean will give better results.

NOTE: Appendix 3 shows a series expansion that may be used successfully for the range where:

$$(I_c + I_3)R_L/E_{cc} < .5$$

$I_c + I_3$ = the final value of the collector current after the step, and E_{cc} = the applied collector supply voltage.

g_m --- No attempt was made here to evaluate this parameter since it does not actually appear in the final expressions, but since g_m is used in the derivation as if it were a constant, errors result.

α_{ce} --- An average value is best but the value at 1 ma. gives satisfactory results as long as great accuracy is not desired. Herein the use of α_{cb} is recommended.

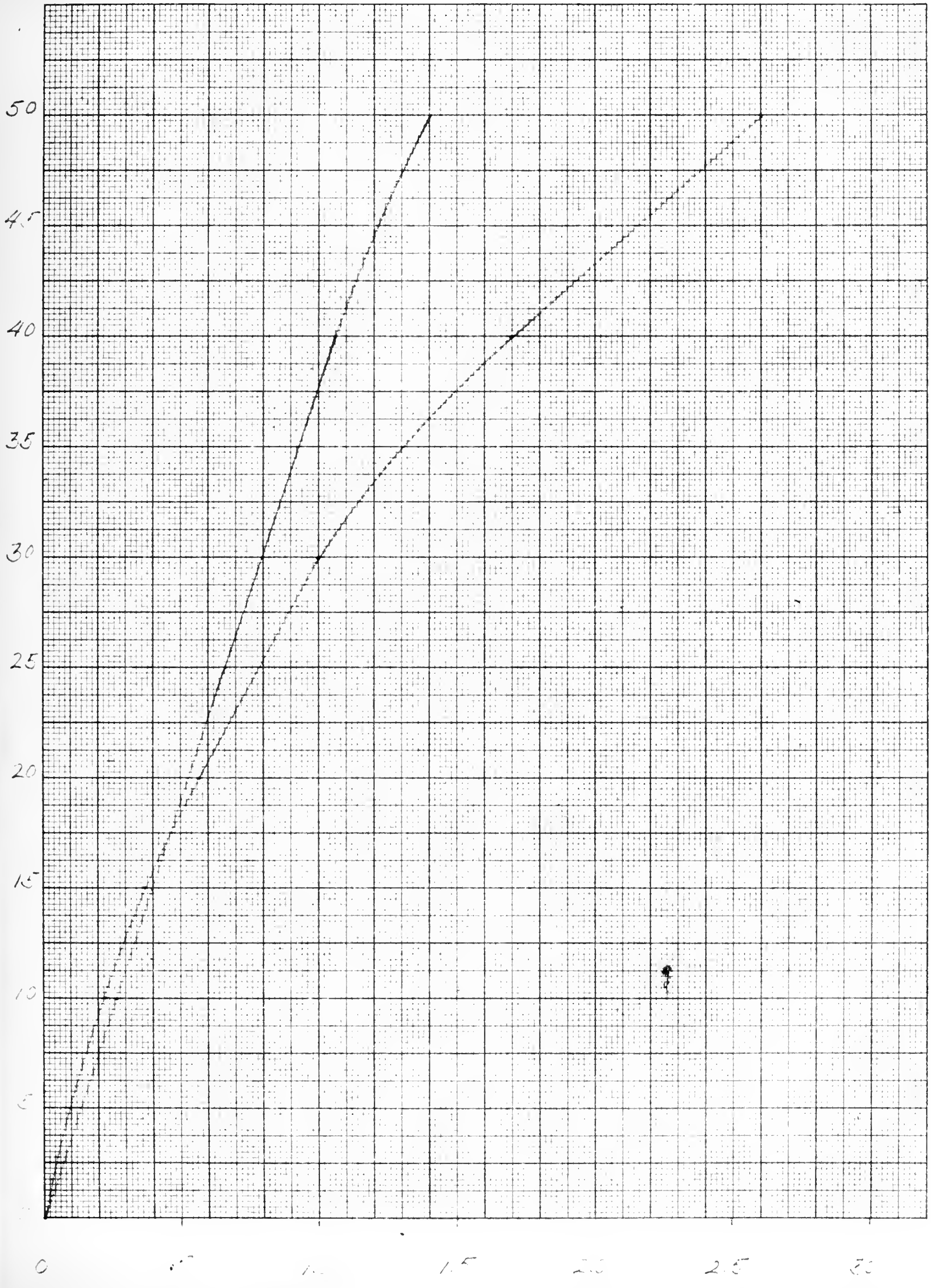
α_{cb} --- An average of the maximum value and a low current value gives the best results. These are small signal AC measurements at about 1 KC. The large signal DC α_{cb} measurements were investigated and found to give even poorer agreement with actual measurements than a simple value of the small signal α_{cb} at 1 ma. and much worse than the average suggested above.

ω_{ce} --- An average here is best as in the α_{cb} considerations but merely the use of the 1 ma. value gives reasonable results. The direct measurement of ω_{cb} was found to give the best results and is recommended.

ω_{cb} --- An average is best also but here again as in the α_{cb} considerations the use of the 1 ma. value gives reasonable results.

5. Transfer Characteristic Linearity

The transfer characteristic linearity also effects the rise and fall time considerations. Figure 13 shows examples of a normal unit, that is, one with linear transfer characteristic and one with very bad α_{cb} falloff at high collector current. This effect in a unit results in very unpredictable operation as a pulse amplifier. Units with non-linear transfer characteristic are not recommended for this application.



I_f in a   Figure

CHAPTER IV

CALCULATIONS USING AN APPROXIMATION TO THE NONLINEAR EQUATIONS

1. Theory

L. J. Giacoletto[3] derives the theoretical equation for collector current from the work of Shockley as:

$$I_{c'} - I_{c's} = I_{c'e'}\ e^{\Lambda V_{e'b'}} + I_{c'c'}\ e^{\Lambda V_{c'b'}}$$

Where the notation is explained in [3], except here small letter subscripts are used instead of capitals as in the reference. This equation is also discussed by J. J. Ebers and J. L. Moll, "Large Signal Behavior of Junction Transistors"[7].

The expressions $V_{e'b'}$ and $V_{c'b'}$ are negative quantities in the sense written here. The normal sense for the quantity $V_{e'b'}$ in this paper is $V_{b'e}$ which is a small positive quantity. The voltage $V_{c'b'}$ is about equal to V_{ce} for the regions under consideration and except at the saturated region is a relatively large negative quantity, always equalling $E_{cc} - i_c R_L$. Hence, since $\Lambda = 38.6$ volts $^{-1}$ at 27°C, the second term on the right in the above equation is normally negligible for the regions here considered and is disregarded.

Rewriting the expression for collector current in the notation of this paper:

$$I_c' - I_{co} = I_o\ e^{V_{b'e}}$$

where:

eI_o = Current that flows in the collector to emitter loop with normal back bias at the collector junction of $E_{cc} > .5$ volt and forward bias at the emitter junction of $1/\Lambda$ volts. This is a very small quantity and hence could not be measured directly without difficulty. Appendix 3 shows a simple method for measuring this quantity. It is interesting to note here that this quantity does not appear in the rise time and fall time formulas, but is necessary when one desires to find the amplitude of voltages and currents concerned with the operation of the circuit.

I_{co} = Collector saturation current when both junctions are biased in the reverse direction.

The method for obtaining the expressions that follow is worked out in detail in Appendix 3. The basic procedure used is as follows:

a. Assume a good approximation of the collector current and solve for the necessary values that would make the applied open circuit voltage a step for the assumed output current. For example: assume $\overline{I}_c = I_3 + I_c\ (1 - e^{-t/T})$ where the notation conforms with that in the Appendix.

b. Set this equal to $I_o\ e^{V_{b'e}/K}$.

c. Solve for $V_{b'e}$ which contains a term such as $\ln \left[1 - \frac{I_c}{I_c + I_3} e^{-t/T} \right]$.

d. Expand the ln term in a series. For a first approximation, keep the first term of this expansion only, keeping more and more terms as the accuracy demands. In Appendix 3., this is worked out for one term and two terms for rise time and one term only for fall time. This was because a method of obtaining approximately the same accuracy for fall time with only one term was found while rise time required two terms.

e. Solve for I_b and ultimately for the applied open circuit step of voltage which is approximated by assuming a $T = \frac{1}{n\omega_{cb}}$ and solving for n such that $\dot{E}(t)$ contains no exponential terms and hence approximates a step to the accuracy of the number of terms of the expansion kept in part d. above.

2. Results Using This Method

The results of the application of this method to numerous assumed conditions of the circuit as derived in Appendix 3 are summarized below:

a. <u>Rise Time</u>

(1) For the approximation (If $\alpha_{cb} R_L C_{b'c} << \frac{1}{\omega_{cb}}$)

For use where $I_c << I_c + I_3$

$$E(t) \doteq \frac{(r_{bb'} + R_b)(I_3 + I_c)}{\alpha_{cb}} + \frac{1}{\Lambda} \ln \left[\frac{I_3 + I_c}{I_o} \right]$$

$$V_{b'e} \doteq \frac{1}{\Lambda} \left[\ln \left(\frac{I_c + I_3}{I_o} \right) \right] - \frac{1}{\Lambda} \left[\frac{I_c}{I_c + I_3} e^{-t/T} \right]$$

$$I_b = \frac{I_c}{\Lambda (R_g + r_{bb'})(I_3 + I_c)} e^{-t/T} + \frac{I_3 + I_c}{\alpha_{cb}}$$

$$\bar{I}_c = I_3 + I_c\,(1 - e^{-t/T})$$

Where:

$$T = \frac{1}{n\,\omega_{cb}}$$

$$n = \frac{\alpha_{cb}}{\Lambda (R_g + r_{bb'})(I_c + I_c)} + 1$$

$$T_R = \frac{2.2}{\dfrac{\alpha_{cb}}{\Lambda (R_g + r_{bb'})(I_3 + I_c)} + 1} \left(\frac{1}{\omega_{cb}} \right)$$

The symbol meanings are as follows:

I_o, I_{co} = defined in the first part of this chapter

$E(t)$ = open circuit applied step of voltage

I_b = total base current

$\bar{I}_c$ = total collector current

I_c = final value of collector current after the transient but not including I_3

I_3 = quiescent collector current minus I_{co}

Λ = 38.6 volts^{-1} at T. 27 C. = q/kT

R_g = generator resistance

$r_{bb'}$ = base lead resistance

α_{cb} = base to collector current gain. See Chapter III on variation of parameters

ω_{cb} = Radian frequency where α_{cb} is .707 of the magnitude of the low frequency value.

It should be noted that if the substitutions of $I_3 + I_c \doteq I_c$ and $\Lambda I_c \doteq g_m$ and $\alpha_{cb}/g_m \doteq r_{b'e}$ are made, this expression reduces to the one in Chapter II, which is a good check on the work thus far. Thus we see that the use of one term of the series approximation is about equivalent to the approach of Chapter II and will be useful at relatively large R_g only. However, even carrying the approach this far gives a more useful result containing an expression for the bias current, etc., and gives a better insight into the meaning of various parameters and where they should be measured.

(2) For the one term approximation (where $C_{b'c}$ is significant)

For use where $I_c << I_c + I_3$

$$E(t) = \frac{(r_{bb'} + R_g)(I_3 + I_c)}{\alpha_{cb}} + \frac{1}{\Lambda} \ln\left[\frac{I_3 + I_c}{I_o}\right]$$

$$V_{b'e} = \frac{1}{\Lambda}\left[\ln\left(\frac{I_3 + I_c}{I_o}\right) - \frac{I_c}{I_c + I_3} e^{-t/T}\right]$$

$$\bar{I}_c = I_3 + I_c\left(1 - e^{-t/T}\right)$$

Where:

$$T = \frac{1}{n\,\omega_{cb}}$$

$$n = \frac{\dfrac{\alpha_{cb}}{\Lambda(R_g + r_{bb'})(I_c + I_3)} + 1}{\alpha_{cb}\,\mu}$$

If: $\left[\dfrac{(I_c + I_3)R_L}{E_{cc}}\right] < .5$

$$\mu = \frac{1}{\alpha_{cb}} + \frac{K_5 R_L \omega_{cb}}{(E_{cc})^{\frac{1}{2}}}\left[1 + .18\,\frac{R_L I_c}{E_{cc}} + \frac{R_L I_3}{2E_{cc}}\right]$$

$K_5 \doteq C_c(V_{ceo})^{\frac{1}{2}}$ $\qquad$ V_{ceo} is the value at which C_c is measured.

Then:

$$T_R = \frac{2.2}{\dfrac{\alpha_{cb}}{\Lambda(R_g + r_{bb'})(I_c + I_3)} + 1}\left[\frac{1}{\omega_{cb}} \quad \frac{\alpha_{cb} K_5 R_L}{(E_{cc})^{\frac{1}{2}}}\left(1 + .18\,\frac{R_L I_c}{E_{cc}} + \frac{R_L I_3}{2E_{cc}}\right)\right]$$

If $\quad .5 < \left[\dfrac{(I_c + I_3)R_L}{E_{cc}}\right]$

Then some other approximation to $C_{b'c}$ should be used such as the one given in Chapter III, hence:

$$u = \left(\frac{1}{\alpha_{cb}} + \omega_{cb} R_L C_{b'c}\right)$$

$$T_R = \frac{2.2}{\dfrac{\alpha_{cb}}{\Lambda(R_g + r_{bb'})(I_c + I_3)} + 1}\left[\frac{1}{\omega_{cb}} + \alpha_{cb} R_L C_{b'c}\right]$$

Where:

$$C_{b'c} \doteq C_c\sqrt{\frac{V_{ceo}}{V_{ce}}}$$

$V_{ceo} = V_{ce}$ where C_c is measured as in Chapter III

$\bar{V}_{ce}$ = Average V_{ce} during the step

E_{cc} = Collector supply voltage

R_L = Collector load resistor.

(3) For the two term approximation (If $\alpha_{cb} R_L C_{b'c} << 1/\omega_{cb}$)

where: $I_c < I_c + I_3$

$$E(t) = \frac{(R_g + r_{bb'})(I_c + I_3)}{\alpha_{cb}} + \frac{1}{\Lambda}\left(\frac{I_3 + I_c}{I_o}\right)$$

$$V_{b'e} = \frac{1}{\Lambda}\left[\ln\frac{I_3 + I_c}{I_o} - \frac{I_c - I_{c2}}{I_c + I_3}e^{-t/T} - \frac{2I_{c2} + (I_c - I_{c2})^2}{2(I_c + I_3)}e^{-2t/T}\right]$$

$$I_b = \frac{I_3 + I_c}{\alpha_{cb}} \quad \frac{1}{\Lambda(I_c + I_3)}\left[(I_c - I_{c2})e^{-t/T} + 2I_{c2}\,e^{-2t/T}\right] + \frac{I_{c2}}{\alpha_{cb}}e^{-2t/T}$$

$$\bar{I}_c = I_3 + (I_c - I_{c2})(1 - e^{-t/T}) + I_{c2}\left(1 - e^{-2t/T}\right)$$

Where:

$$T = \frac{1}{n\,\omega_{cb}}$$

n = same as (1) and (2)

$$I_{c2} = K_4 - K_4\left[1 - \left(\frac{I_c}{K_4}\right)^2\right]^{\frac{1}{2}}$$

$$K_4 = 2I_c + I_3 + \frac{\Lambda(R_g + r_{bb'})}{\alpha cb}(I_c + I_3)^2$$

$$T_R = \frac{\ln\left[9\,\frac{I_c - .9I_{c2}}{I_c - .1I_{c2}}\right]}{\frac{\alpha_{cb}}{\Lambda(R_g + r_{bb'})(I_c + I_3)}+1}\left(\frac{1}{\omega_{cb}}\right), \quad I_{c2} << I_c$$

(4) For the two term approximation (where $C_{b'c}$ is significant) where $I_c < I_c + I_3$

$E(t)$, $V_{b'e}$, I_b, and $\bar{I}_c$ remain the same as (3).

If: $\left[\frac{(I_c + I_3)R_L}{E_{cc}}\right] < .5$

$$T_R \doteq \frac{\ln\left[9\,\frac{I_c - .9I_{c2}}{I_c - .1I_{c2}}\right]}{\frac{\alpha_{cb}}{\Lambda(R_g + r_{bb'})(I_c + I_3)}+1}\left[\frac{1}{\omega_{cb}} + \frac{\alpha_{cb}I_5R_L}{(E_{cc})^{\frac{1}{2}}}\left(1 + .18\,\frac{R_LI_c}{E_{cc}} + \frac{R_LI_3}{2E_{cc}}\right)\right]$$

$K_5 \doteq C_c(V_{ceo})^{\frac{1}{2}}$ as before

If $\quad .5 < \left[\frac{(I_c \quad I_3)R_L}{E_{cc}}\right]$

$$T_R = \frac{\ln\left[9\,\frac{I_c - .9I_{c2}}{I_c - .1I_{c2}}\right]}{\frac{\alpha_{cb}}{\Lambda(R_g + r_{bb'})(I_c + I_3)}+1}\left[\frac{1}{\omega_{cb}} + \alpha_{cb}\,R_LC_{b'c}\right]$$

$$C_{b'c} = C_c \sqrt{\frac{V_{ceo}}{\overline{V}_{ce}}} \qquad \text{as before}$$

b. __Fall Time:__

(1) For the one term approximation (If $\alpha_{cb} R_L C_{b'c} \ll 1/\omega_{cb}$)

where $I_c < I_4 + I_c$

$$E(t) = \frac{(I_c + I_4)(R_g + r_{bb'})}{\alpha_{cb}} + \frac{1}{\Lambda} \ln \left(\frac{I_4 + I_c}{I_o} \right)$$

$$V_{b'e} = \frac{1}{\Lambda} \left[\ln \frac{I_4}{I_o} + \frac{2I_c}{2I_4 + I_c e^{-t/T}} e^{-t/T} \right]$$

$$I_b = \frac{I_4}{\alpha_{cb}} - \frac{2I_c}{\Lambda (R_g + r_{bb'})(2I_4 + I_c e^{-t/T})} e^{-t/T}$$

$$\overline{I}_c = I_4 + I_c e^{-t/T}$$

where:

$$T_1 = \frac{1}{n_1 \omega_{cb}}$$

$$T_2 = \frac{1}{n_2 \omega_{cb}}$$

$$n_1 = \frac{2 \alpha_{cb}}{\Lambda (R_g + r_{bb'})(2I_4 + .95I_c)} + 1$$

$$n_2 = \frac{2 \alpha_{cb}}{\Lambda (R_g + r_{bb'})(2I_4 + .1I_c)} + 1$$

$$T_R = \left[\frac{2.3}{\dfrac{2\alpha_{cb}}{\Lambda(R_g + r_{bb'})(2I_4 + .1I_c)} + 1} - \frac{.104}{\dfrac{2\alpha_{cb}}{\Lambda(R_g + r_{bb'})(2I_4 + .95I_c)} + 1}\right]\frac{1}{\omega_{cb}}$$

The symbols here have the same meanings as in the rise time considerations with I_4 here corresponding to I_3 in the rise time formulae.

(2) For the one term approximation (where $C_{b'c}$ is significant) where $I_c < I_4 + I_c$

$E(t)$, $V_{b'e}$, I_b, and $\bar{I}_c$ are unchanged.

If: $\left[\dfrac{(I_c + I_4)R_L}{E_{cc}}\right] < .5$

$$T_R = \left[\frac{2.3}{\dfrac{2\alpha_{cb}}{\Lambda(R_g + r_{bb'})(2I_4 + .1I_c)} + 1} - \frac{.104}{\dfrac{2\alpha_{cb}}{\Lambda(R_g + r_{bb'})(2I_4 + .95I_c)} + 1}\right] \cdot \left[\frac{1}{\omega_{cb}} + \frac{\alpha_{cb}K\,R_L}{(E_{cc})^{\frac{1}{2}}}\left(1 + \frac{.5R_L(I_c + I_4)}{E_{cc}}\right)\right]$$

If: $.5 < \left[\dfrac{(I_c \quad I_4)R_L}{E_{cc}}\right]$

$$T_R = \left[\frac{2.3}{\dfrac{2\alpha_{cb}}{\Lambda(R_g + r_{bb'})(I_4 + .1I_c)} + 1} - \frac{.104}{\dfrac{2\alpha_{cb}}{\Lambda(R_g + r_{bb'})(I_4 + .95I_c)} + 1}\right]\left(\frac{1}{\omega_{cb}} + \alpha_{cb}R_L C_{b'c}\right)$$

where:

$$C_{b'c} = 1.5C_c\sqrt{\frac{V_{ceo}}{\bar{V}_{ce}}}$$

3. Comments on the Results

The above approach was used in an attempt to obtain correlation with theory to within 10 per cent. The results and a comparison with measured values are the subject of Chapter V.

In general, if closer correlation with theory is desired, it could be achieved by considering more terms of the various series and using the same approach to solve for the desired result, however, as can be seen from the two term approximation for rise time, this leads to very cumbersome expressions at best.

This same approach can easily be extended to include the grounded base and grounded collector stages. Also by considering the approximations to the basic equation that may be employed in the saturated region, this method may be extended to include this region for rise time and fall time. To employ a similar attack on storage time seems possible, but no attempt was made in this study to consider storage time.

CHAPTER V

CORRELATION WITH THEORY

1. Introduction

No attempt was made to correlate voltage gain with theory, only rise time and fall time being considered.

The basic procedure followed was first to compare measurements on a number of units for the case where there were the least number of variables, namely, where $C_{b'c}$ was not important, where R_g was large enough to assume a step of current at the base, using constant drive and I_3 essentially zero. Then with this comparison as a starting point, typical units are used to compare theoretical with measured values for a wide range of variation of R_g, R_L, I_3 and I_c (final value).

2. Discussion

a. Figure 14 shows a plot of T_R against $1/\omega_{cb}$ which is the only variable remaining when the conditions described above are imposed. Twenty-seven units are plotted with a range of α_{cb} (average) from 25 to 133 and a spread of ω_{cb} of nearly two octaves. Agreement here is within 10% for all units checked. The line is theory, the dots measured.

b. Figure 15 is a similar plot for fall time using the same units. Agreement here is only within 20% for all units, but this is far better than that obtained when it is assumed that rise time and fall time are about the same, which even under these conditions will give errors as high as 200%. The line is theory, the dots measured.

c. Figure 16 shows the variation of rise time with R_g over the

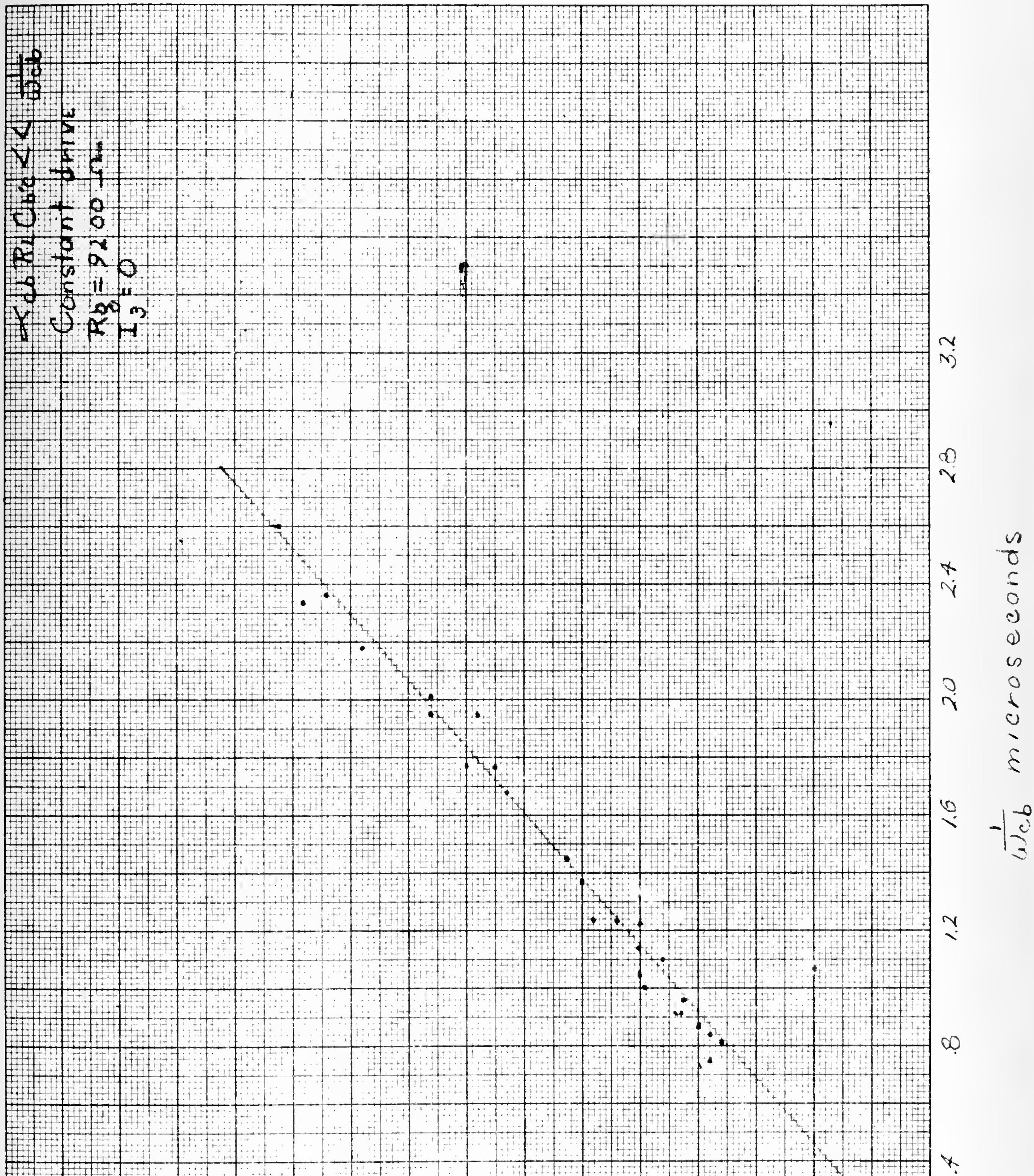

Constant drive
Rb = 9200 Ω
I3 = 0
1/ωcb microseconds
TR microseconds
.4
.8
1.2
1.6
2.0
2.4
2.8
3.2
0
1
2
3
4
5
6

Figure 14

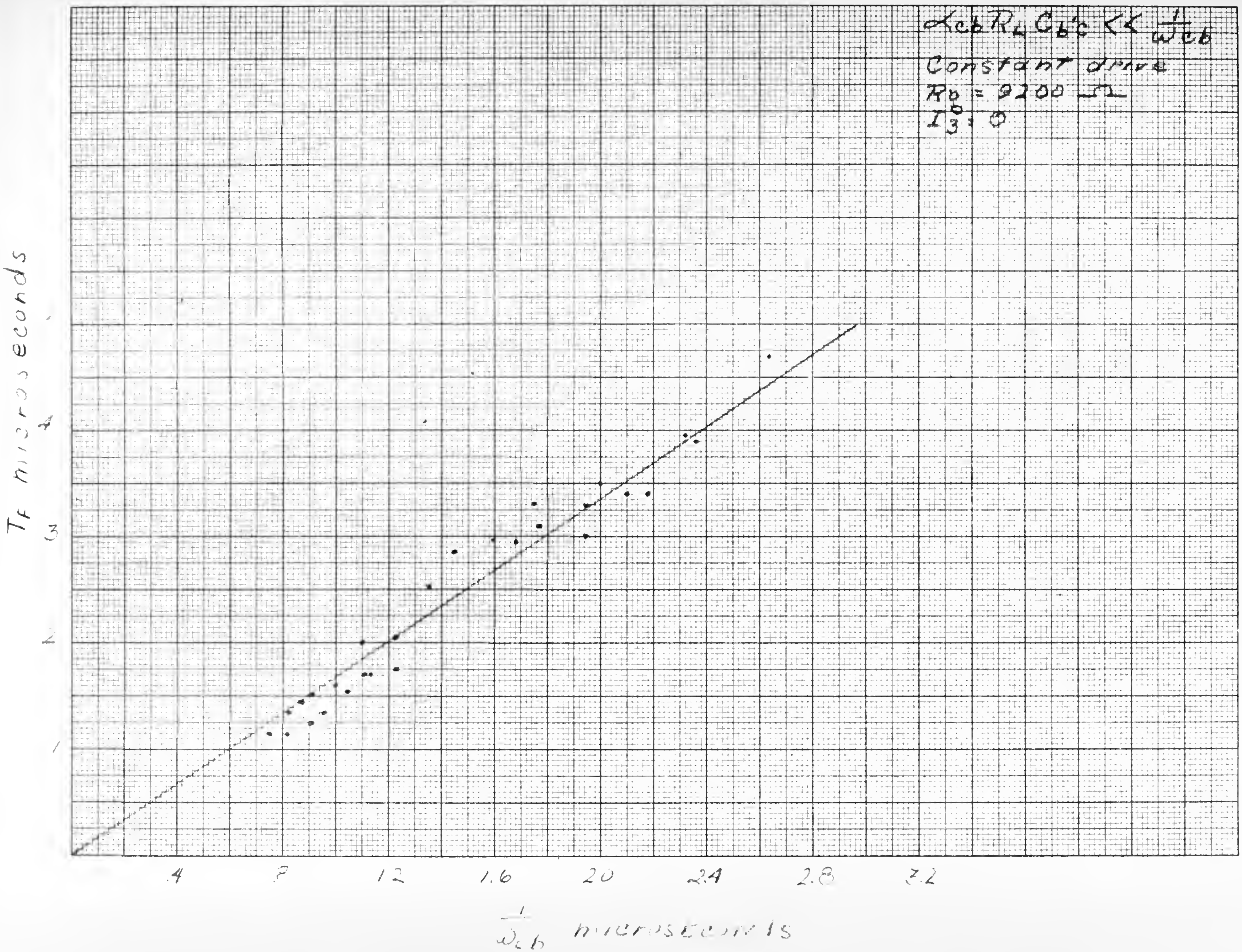
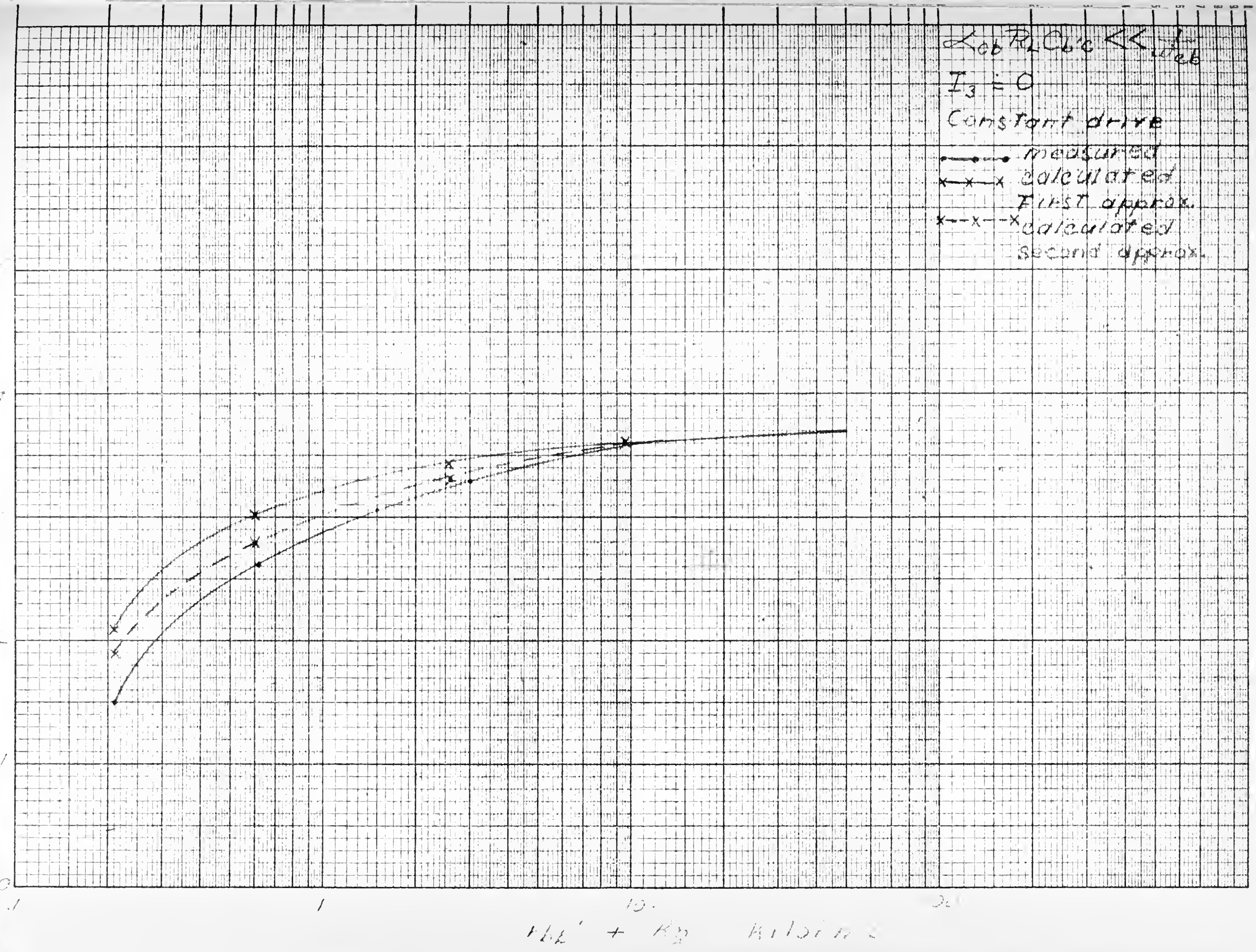
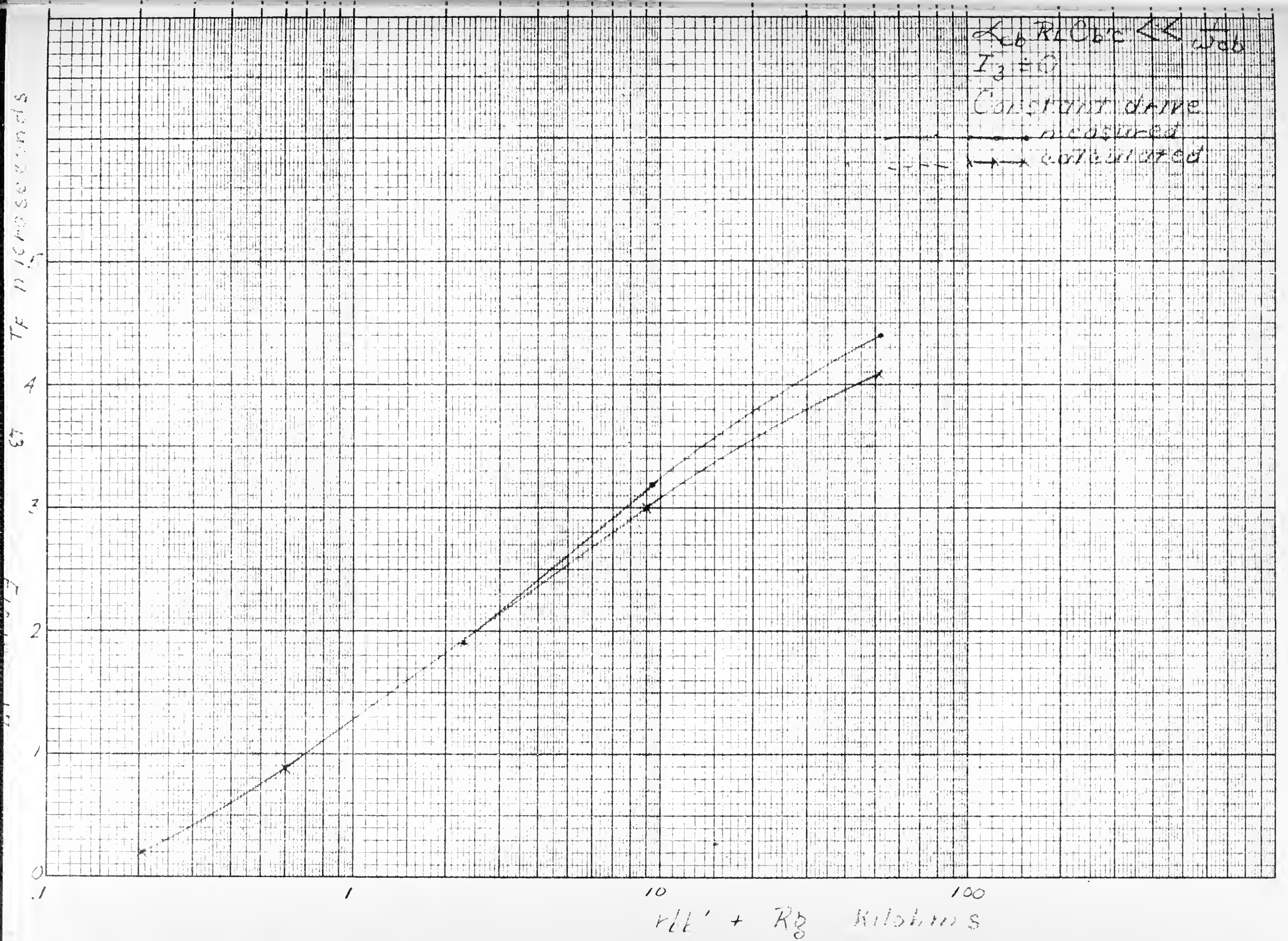

$\alpha_{cb} R_L C_{b'c} << \frac{1}{\omega_{cb}}$
Constant drive
$R_0 = 9200\ \Omega$
$I_3 = 0$
T_F microseconds
1
2
3
4
.4
.8
1.2
1.6
2.0
2.4
2.8
3.2
$\frac{1}{\omega_{cb}}$ microseconds

Figure 15



$\omega_{eb} R_L C_{b'c} << \frac{1}{\omega_{eb}}$
$I_3 = 0$
Constant drive
measured
calculated
First approx.
calculated
second approx.


$L_{cb} R_L C_{b'c} \ll \frac{1}{\omega_{ab}}$
$I_3 = 0$
Constant drive
measured
calculated
T_F microseconds
43
0
1
2
3
4
.1
1
10
100
$r_{bb'} + R_g$ Kilohms

range from R_g = 93 ohms to R_g 50000 ohms for a typical unit and compares both the one term approximation and the two term approximation with actual values. At very low R_g, the agreement with theory is only within 20% for this unit. This represents the maximum observed error at low R_g.

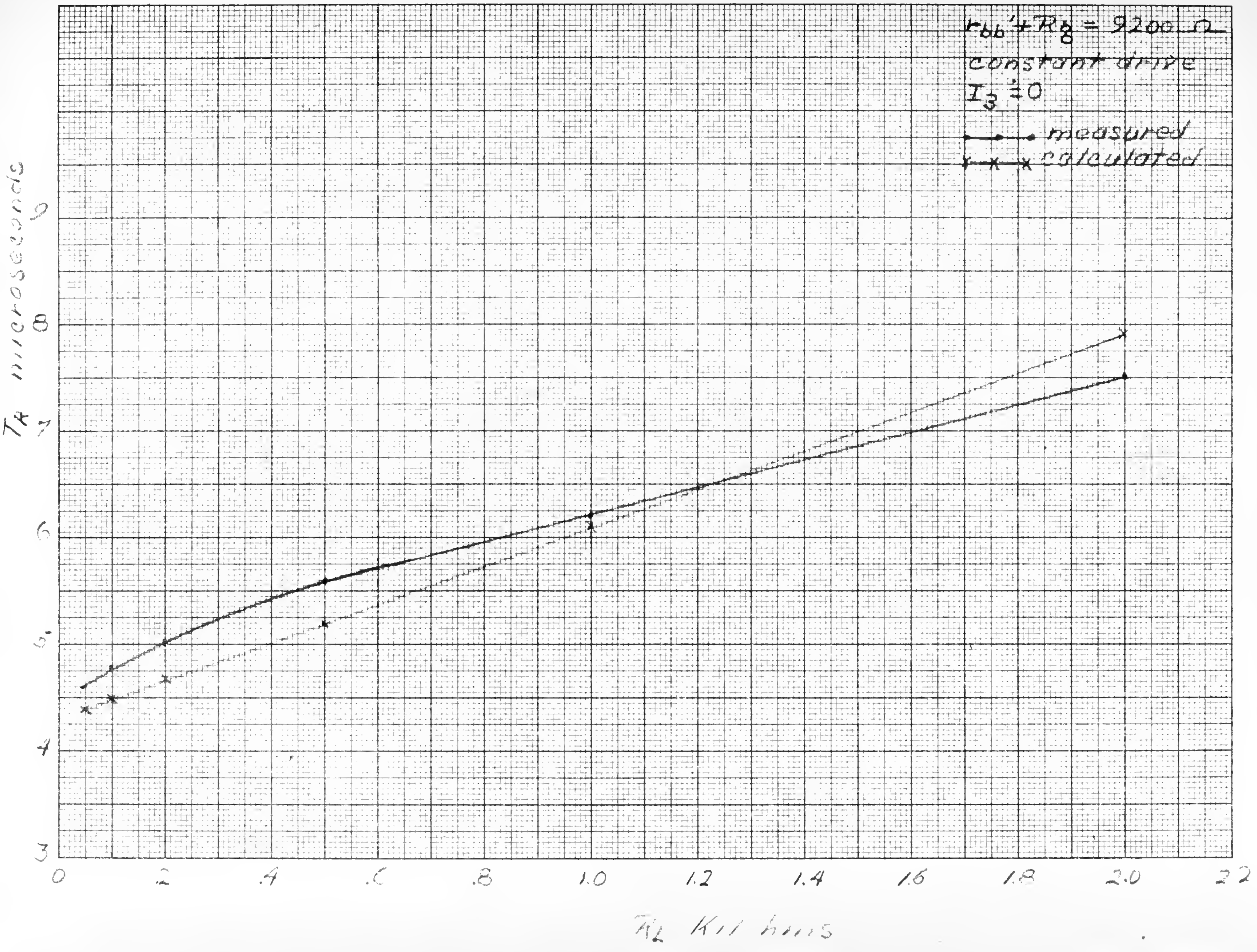
d. Figure 17 shows a similar plot for fall time. Here correlation is very good over the whole range of variation considered. Per cent errors were fairly large at very low R_g for some units, but measurements at tenths of microseconds are subject to larger per cent errors also.

e. Figures 18 and 19 show variation of fall time and rise time with R_L over the range from R_L = 50 ohms to R_L = 2000 Ω and agree well with theory for the range considered.

f. Figure 20 shows variation of fall time and I_3. Agreement here is good also. Under the conditions imposed, fall time is essentially constant with I_3 except at very small signals, which are not considered here.

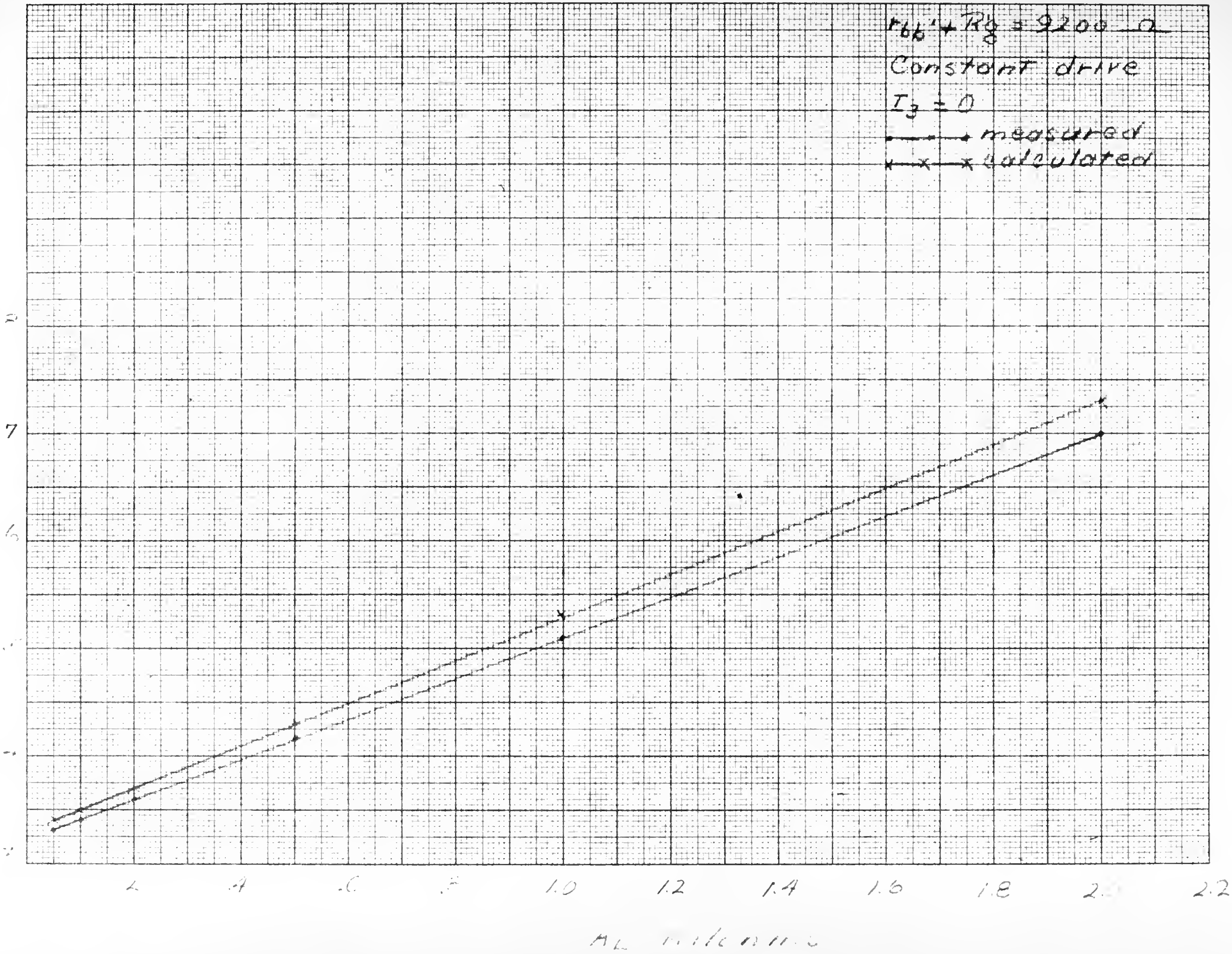
g. Figure 21 shows the variation of rise time with the final value of the collector current transient. Agreement here is erratic because an average value of α_{cb} and ω_{cb} were used, and at low I_c, these are least accurate. This is essentially a plot of variation of rise time with drive.

h. Figure 22 shows the variation of fall time with drive.



$r_{bb'} + R_g = 9200\ \Omega$
constant drive
$I_3 \doteq 0$
measured
calculated
T_R microseconds
3
4
5
6
7
8
9
0
.2
.4
.6
.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
R_L K ohms

Figure 16



$r_{bb'} + R_g = 9200\ \Omega$
Constant drive
$I_3 = 0$
measured
calculated
7
1.0
1.2
1.4
1.6
1.8
2.2

Figure 19

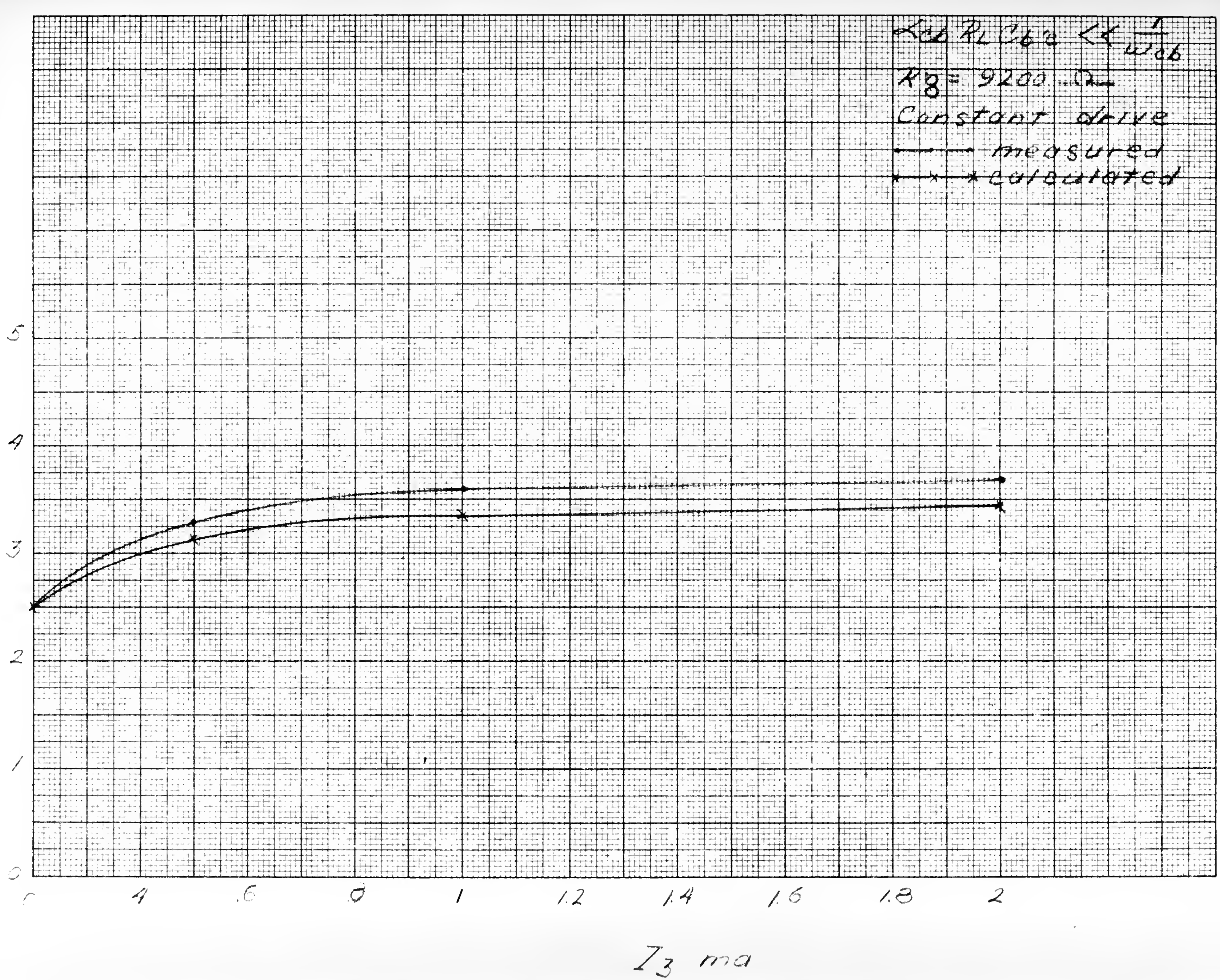

$\omega_{cb} R_L C_{b2} << \frac{1}{\omega_{cb}}$
$R_g = 9200\ \Omega$
Constant drive
measured
calculated
T_F microseconds
0
1
2
3
4
5
0
.4
.6
.8
1
1.2
1.4
1.6
1.8
2
I_3 ma

Figure 20

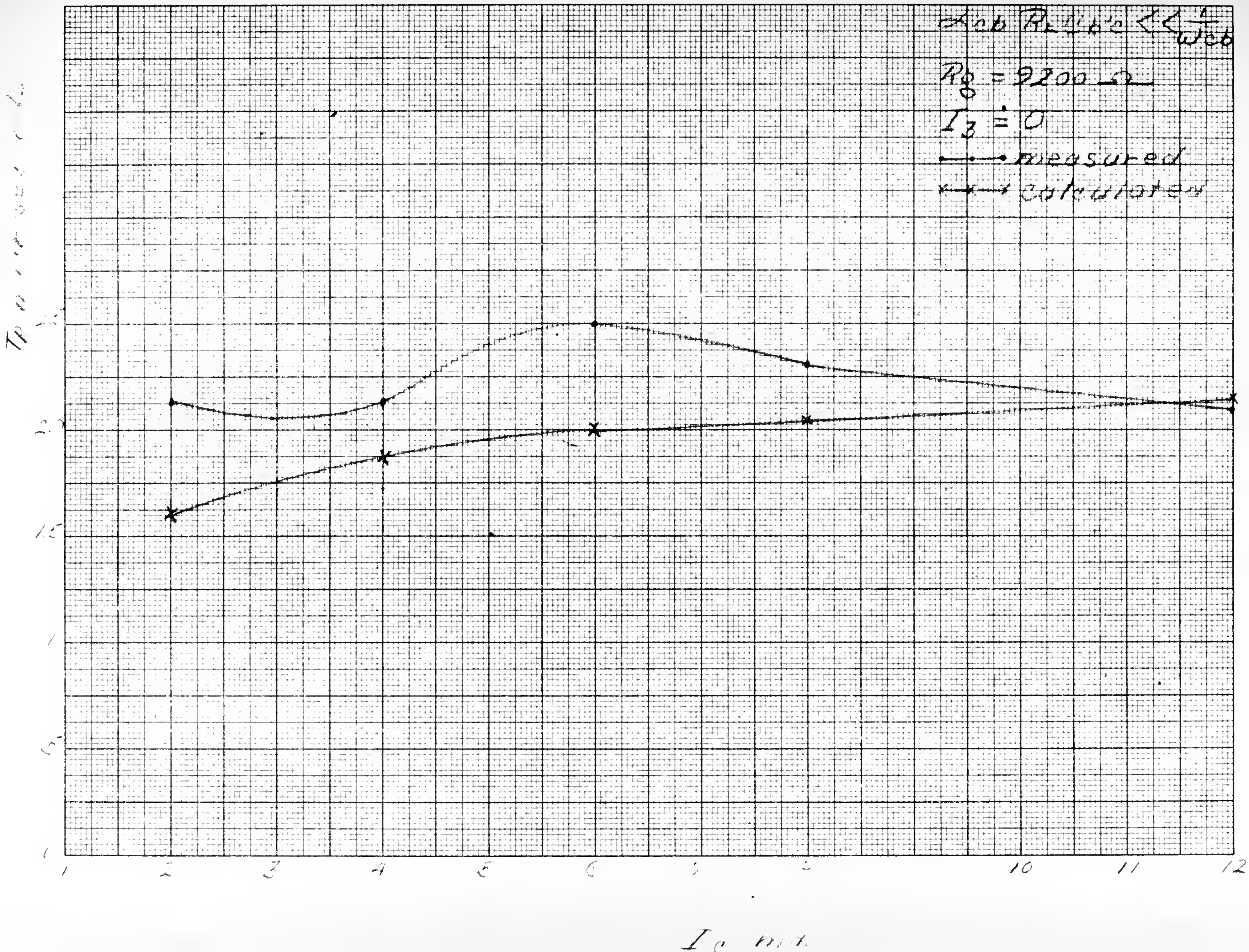
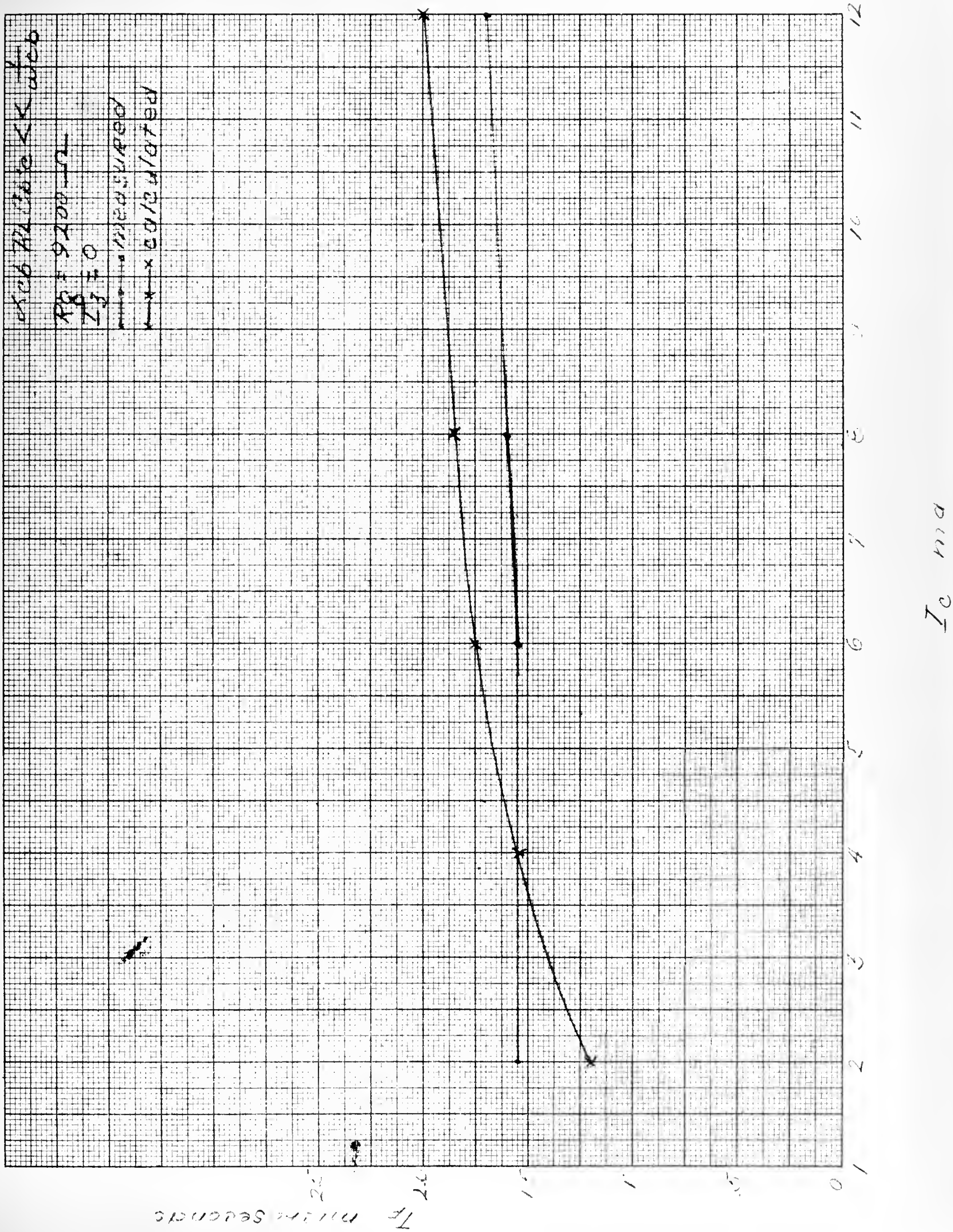

$R_g = 9200\ \Omega$
$I_3 \doteq 0$
measured
calculated
1
2
3
4
5
6
10
11
12
I_c ma

Figure 21



$r_{cb}\, r_L\, C_{b'e} << \frac{1}{\omega_{cb}}$
$R_g = 9200\ \Omega$
$I_B = 0$
measured
calculated
I_c ma

Figure 22.

CHAPTER VI

CONCLUSIONS AND COMMENTS

1. The following conclusions were drawn from this study:

a. The most important single parameter of the transistor as used for pulse amplifier service in the grounded emitter configuration is ω_{cb}. This is currently determined almost entirely by diffusion time, but when commercial units are available where this factor is greatly reduced, transition capacities may be most significant.

b. Collector capacity is significant under certain conditions (where $\alpha_{cb}R_LC_{b'c}$ becomes significant compared to $1/\omega_{cb}$) and seems destined to become the final limiting factor on frequency response of units of the future, at least when used in the grounded emitter configuration.

c. The value of $r_{bb'}$ becomes significant to some extent at low values of R_g and should not be overlooked.

d. The most important external circuit parameters are generator resistance, R_g and load resistance R_L. Judicious use of proper values here will greatly enhance circuit performance. It should be noted in this connection that in a two stage cascade amplifier with low values of R_L, driven from a low source impedance, it is possible to have the total rise time and the total fall time about equal the rise time of one stage if the operating point is chosen so that the quiescent collector current is essentially zero.

e. The expressions derived are large signal expressions and should be used for very small signals with caution.

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APPENDIX I

TRANSIENT ANALYSIS OF THE LINEAR EQUIVALENT CIRCUIT USING LAPLACE TRANSFORMS, AND VARIOUS APPROXIMATIONS

To find the transient response of the approximate transistor using the short circuit case suggested in Chapter II, making use of Laplace Transform notation since a linear relationship is assumed:

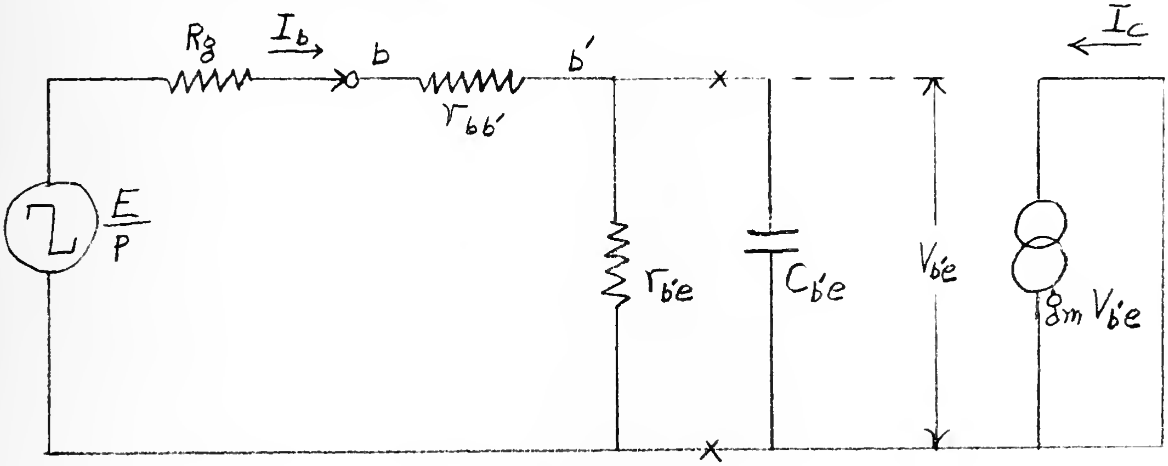

Figure 23

If a step of voltage is applied, using Thevenin's Theorem at xx replace by:

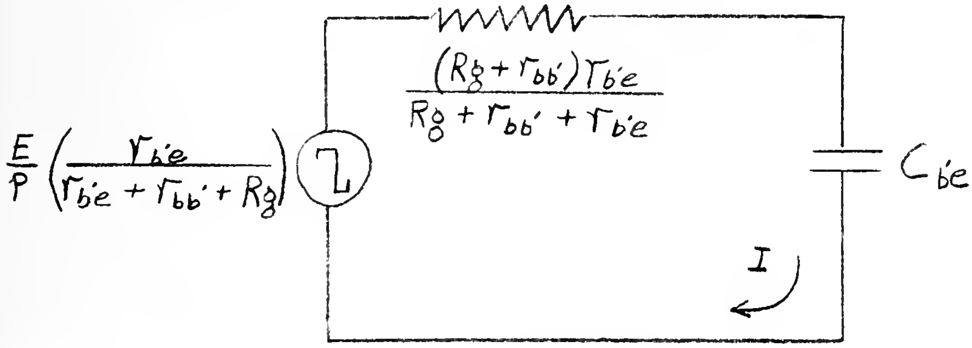

Figure 24

Let: $\dfrac{(R_g + r_{bb'})r_{b'e}}{R_g + r_{bb'} + r_{b'e}} = R_1$

and: $\dfrac{E}{p}\left(\dfrac{r_{b'e}}{R_g + r_{bb'} + r_{b'e}}\right) = \dfrac{E_1}{p}$

then: $\left(R_1 + \dfrac{1}{C_{b'e}\,p}\right) \quad I = \dfrac{E_1}{p}$

where: $p = \sigma + j\omega$, the normal Laplace operator

then:

$$I = \frac{E_1}{p\left(\dfrac{R_1 C_{b'e} p + 1}{C_{b'e} p}\right)}$$

$$V_{b'e} = \frac{I}{C_{b'e} p}$$

$$= \frac{E_1}{R_1 C_{b'e}} \quad \frac{1}{p\left(p + \dfrac{1}{R_1 C_{b'e}}\right)}$$

or

$$V_{b'e} = E_1\left(1 - e^{-\frac{t}{R_1 C_{b'e}}}\right)$$

$$I_c \doteq g_m V_{b'e}$$

Therefore:

$$I_c = \frac{g_m r_{b'e}\, E}{R_g + r_{bb'} + r_{b'e}}\left[1 - e^{-\left(1 + \frac{r_{b'e}}{R_g + r_{bb'}}\right)\frac{t}{r_{b'e} C_{b'e}}}\right]$$

since:

$$g_m r_{b'e} \doteq \alpha_{cb}\,, \quad \frac{1}{r_{b'e} C_{b'e}} \doteq \omega_{cb}$$ See Appendix 2

$$I_c = \frac{\alpha_{cb} \; E}{R_g + r_{bb'} + r_{b'e}} \left[1 - e^{-\left(1 + \frac{r_{b'e}}{R_g + r_{bb'}}\right) \omega_{cb} \, t} \right]$$

If a step of current I/p is applied with shunt conductance G_g, the results are the same except the voltage is replaced by current and R_g - hence;

$$I_c = \frac{\alpha_{cb} R_g \; I}{r_{bb'} + r_{b'e} + R_g} \left[1 - e^{-\left(1 + \frac{r_{b'e}}{r_{bb'} + R_g}\right) \omega_{cb} \, t} \right]$$

In this simple approach, $T_R = T_F$ and is found as follows from the 10 per cent to the 90 per cent point, calling t_1 the time associated with the 10% point and t_2 the time associated with the 90% point:

$$T_R = t_2 - t_1$$

at t_1 - - I_c = .1 of final value

$$I_c = \frac{.1 \, \alpha_{cb} \; E}{R_g + r_{bb'} + r_{b'e}}$$

hence:

$$.9 = e^{-\left(1 + \frac{r_{b'e}}{r_{bb'} + R_g}\right) \omega_{cb} \, t_1}$$

$$\ln 1.11 = \left(1 + \frac{r_{b'e}}{r_{bb'} + R_g}\right) \omega_{cb} \, t_1$$

therefore:

$$t_1 = \frac{1}{\omega_{cb}} \left(\frac{1}{1 + \frac{r_{b'e}}{r_{bb'} + R_g}} \right) \ln 1.11$$

Similarly:

$$t_2 = \frac{1}{\omega_{cb}} \left(\frac{1}{1 + \frac{r_{b'e}}{r_{bb'} + R_g}} \right) \ln 10$$

Hence:

$$T_R = t_2 - t_1 = \frac{2.2}{\omega_{cb}} \left(\frac{1}{1 + \frac{r_{b'e}}{r_{bb'} + R_g}} \right)$$

Using the method suggested in Chapter II for treating $C_{b'c}$, since capacitors in parallel add, $C_{b'c}$ multiplied by the voltage gain of the intrinsic transistor adds to $C_{b'e}$. This only changes the time constant, hence:

$$\frac{r_{b'e} + r_{bb'} + R_g}{r_{b'e}(r_{bb'} + R_g)(C_{b'e} + g_m R_L C_{b'c})}$$

since:

$$\frac{1}{r_{b'e} C_{b'e}} \doteq \omega_{cb} \quad , \quad g_m r_{b'e} \doteq \alpha_{cb} \text{ as before}$$

the time constant becomes:

$$\frac{\frac{r_{b'e}}{r_{bb'} + R_g} + 1}{\frac{1}{\omega_{cb}} + \alpha_{cb} R_L C_{b'c}}$$

The expression for I_c is unchanged except for this time constant: therefore:

$$T_R = 2.2 \left[\frac{1}{1 + \frac{r_{b'e}}{r_{bb'} + R_g}} \right] \left(\frac{1}{\omega_{cb}} + \alpha_{cb} R_L C_{b'c} \right)$$

Appendix 2

DERIVATIONS OF α_{cb}, α_{ce}, ω_{cb}, ω_{ce} USING THE HYBRID - PI CIRCUIT

If $\alpha_{cb}(\omega)$ is defined as the short circuit current gain of the grounded emitter stage.

$$\alpha_{cb}(\omega) = -\frac{y_m - y_{b'c}}{y_{b'e} + y_{b'c}} = \frac{(g_m - g_{b'c}) - j\omega C_{b'c}}{(g_{b'e} + g_{b'c}) + j\omega(C_{b'e} + C_{b'c})}$$

$$= -\frac{(g_m - g_{b'c})(g_{b'e} + g_{b'c}) - \omega^2 C_{b'c}(C_{b'e} + C_{b'c}) - j\omega\left[C_{b'c}(g_{b'e} + g_{b'c}) - (g_m - g_{b'c})(C_{b'e} + C_{b'c})\right]}{(g_{b'e} + g_{b'c})^2 + \omega^2 (C_{b'e} + C_{b'c})^2}$$

Define ω_{cb} as the frequency at which the magnitude of $\alpha_{cb}(\omega)$ is $1/\sqrt{2}$ times the low frequency value, α_{cb}.

hence:

$$\alpha_{cb}(\omega) = \alpha_{cb}\left\{\frac{\left[1 - \frac{\omega^2 C_{b'c}(C_{b'e} + C_{b'c})}{(g_m - g_{b'c})(g_{b'e} + g_{b'c})}\right] - j\omega\left[\frac{C_{b'c}}{g_m - g_{b'c}} - \frac{C_{b'e} + C_{b'c}}{g_{b'e} + g_{b'c}}\right]}{1 + \omega^2\left(\frac{C_{b'e} + C_{b'c}}{g_{b'e} + g_{b'c}}\right)^2}\right\}$$

where:

$$\alpha_{cb} = -\frac{g_m - g_{b'c}}{g_{b'e} + g_{b'c}} \doteq \frac{g_m}{g_{b'e}} \doteq \frac{\Delta I_c}{\Delta I_b} = \frac{I_c}{I_b}$$

Therefore: ω_{cb} determined by the relation:

$$\left[1 - \frac{\omega_{cb}^2 C_{b'c}(C_{b'e} + C_{b'c})}{(g_m - g_{b'c})(g_{b'e} + g_{b'c})}\right]^2 + \omega_{cb}^2\left[\frac{C_{b'c}}{g_m - g_{b'c}} - \frac{C_{b'e} + C_{b'c}}{g_{b'e} + g_{b'c}}\right]^2 =$$

$$= \frac{1}{2}\left[1+\omega_{cb}^{2}\left(\frac{C_{b'e}+C_{b'c}}{g_{b'e}+g_{b'c}}\right)^{2}\right]^{2}$$

or:

$$1-\omega_{cb}^{2}\left[8\,\frac{C_{b'c}(C_{b'e}+C_{b'c})}{(g_m-g_{b'c})(g_{b'e}+g_{b'c})}-2\,\frac{C_{b'c}^{2}}{(g_m-g_{b'c})^{2}}\right]+$$

$$\omega_{cb}^{4}\left[\frac{2\,C_{b'c}(C_{b'e}+C_{b'c})^{2}}{(g_m+g_{b'c})^{2}(g_{b'e}+g_{b'c})^{2}}-\frac{(C_{b'e}+C_{b'c})^{4}}{(g_{b'e}+g_{b'c})^{4}}\right]=0$$

Since:

$$g_{b'c} <<< g_m$$

$$g_{b'c} <<< g_{b'e}$$

$$C_{b'c} <<< C_{b'e}$$

This equation becomes:

$$1-8\,\frac{C_{b'c}+C_{b'e}}{g_m g_{b'e}}\,\omega_{cb}^{2}-\left(\frac{C_{b'e}}{g_{b'e}}\right)^{4}\omega_{cb}^{4}=0$$

From which:

$$\omega_{cb}^{2}=\frac{\sqrt{\left(\frac{C_{b'e}}{g_{b'e}}\right)^{2}+16\left(\frac{C_{b'c}}{g_m}\right)^{2}}-4\,\frac{C_{b'c}}{g_m}}{\left(\frac{C_{b'e}}{g_{b'e}}\right)^{3}}$$

If:

$$\frac{C_{b'e}}{g_{b'e}} >> 16\,\frac{C_{b'c}}{g_m}$$

Then:

$$\omega_{cb} \doteq \frac{g_{b'e}}{C_{b'e}} \doteq \frac{1}{(\alpha_{cb}+1)}\,\frac{2\,D_p}{W_b^{2}} \doteq \frac{1}{\alpha_{cb}}\,\frac{2\,D_p}{W_b^{2}}$$

Similarly, solving for $\alpha_{ce}(\omega)$ to the intrinsic base; if $\alpha_{ce}(\omega)$ is defined as the short circuit current gain of the grounded base stage.

$$\alpha_{ce}(\omega) \doteq \frac{-y_m - y_{ce}}{y_{b'e} + y_m + y_{ce}} = \frac{-g_m - g_{ce}}{(g_{b'e} + g_m + g_{ce}) + j\omega C_{b'e}}$$

$$= -\frac{(g_m + g_{ce})(g_{b'e} + g_m + g_{ce}) + j\,\omega(C_{b'e}\,g_m + C_{b'e}\,g_{ce})}{(g_{b'e} + g_m + g_{ce})^2 + \omega^2\, C_{b'e}^{\;2}}$$

Define the ω_{ce} as the frequency at which the magnitude of $\alpha_{ce}(\omega)$ is $1/\sqrt{2}$ times the low frequency value, α_{ce}

hence:

$$\alpha_{ce}(\omega) = \alpha_{ce}\left[\frac{1 - \dfrac{j\omega C_{b'e}}{(g_{b'e} + g_m + g_{ce})}}{1 + \left(\dfrac{C_{b'e}}{g_{b'e} + g_m + g_{ce}}\right)^2}\right]$$

Where:

$$\alpha_{ce} = \frac{-(g_m + g_{ce})}{g_{b'e} + g_m + g_{ce}} \doteq -\frac{g_m}{g_{b'e} + g_m} \doteq \frac{\Delta I_c}{\Delta(I_b + I_c)} = \frac{I_c}{I_e}$$

Therefore: ω_{ce} derived from the relation:

$$1 + \omega_{cb}^2 \left(\frac{C_{b'e}}{g_{b'e} + g_m + g_{ce}}\right)^2 = \frac{1}{2}\left[1 + \left(\frac{\omega_{cb}\, C_{b'e}}{g_{b'e} + g_m + g_{ce}}\right)^2\right]^2$$

or:

$$1 - \omega_{ce}^4 \left(\frac{C_{b'e}}{g_{b'e} + g_m + g_{ce}}\right)^4 = 0$$

$$\omega_{ce} \doteq \frac{g_{b'e} + g_m + g_{ce}}{C_{b'e}} \doteq \frac{g_{b'e} + g_m}{C_{b'e}} \doteq \frac{2\,D_p}{W_b^2}$$

D. Haneman, "Expressions for the α Cut-Off Frequency in Junction Transistors"[9] points out that due to the approximations by some authors, this expression is in error. Actually:

$$\omega_{ce} = \frac{K\,D_p}{W_b^{\,2}}$$

where:

$K = 2.43$ for $\frac{W_b}{L_b} = 0$ and larger values for $\frac{W_b}{L_b} > 0$.

Haneman's remarks apply to the hybrid - pi circuit. Also, this was worked out for the intrinsic base disregarding $r_{bb'}$. ω_{ce} will be a function of $r_{bb'}$ as measured, since this measurement can only be made at the extrinsic base. In this connection see R. L. Pritchard, "Frequency Variation of Junction Transistor Parameters"[8]. Thus, to use the ω_{ce} measurement in the grounded emitter calculations, the measurement would have to be corrected for $r_{bb'}$ and then related to ω_{cb} in the formulas by the expression:

$$\frac{\omega_{ce}}{\omega_{cb}} \doteq 1.2\,\alpha_{cb}, \text{ at } \frac{W_b}{L_b} = 0, \text{ etc.}$$

For this reason, in this paper, ω_{cb} was measured directly throughout and used that way. This writer believes this will give generally better results.

A check of the values of ω_{cb} measured directly and as calculated was made using curves to correct for $r_{bb'}$ and then using the expression:

$$\omega_{cb} = \frac{\omega_{ce}}{1.2\,\alpha_{cb}}$$

The values agreed within 10% for all units provided the readings

were all taken at the same operating point. The disagreement was believed mainly due to $\dot{W}_b/L_b > 0$.

APPENDIX III

TRANSIENT ANALYSIS USING AN APPROXIMATION TO THE NON-LINEAR EQUATIONS

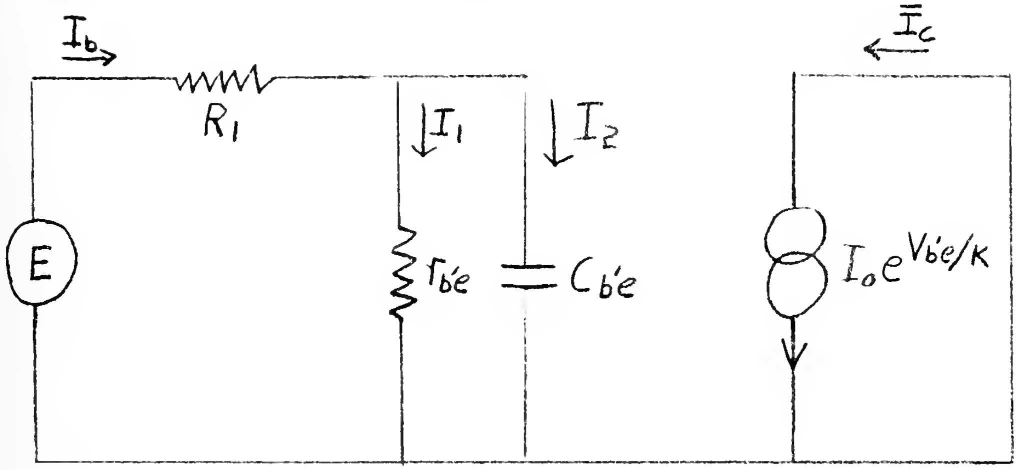

Figure 25

$R_1 = r_{bb'} + R_g$

R_g = Generator Resistance

Output Short Circuited

RISE TIME

Case 1:

Assume $\quad \overline{I}_c = I_3 + I_c \quad (1 - e^{-t/T}) = I_o \quad e^{V_{b'e}/K}$

where $K = \dfrac{1}{\Lambda} = \dfrac{kT}{q} = \dfrac{1}{38.6}$ volts^{-1} at $T = 27^oC$

k = Boltzmann's constant

T = Temperature in ^{o}Kelvin

q = Charge of the minority carrier

I_o can best be obtained by the use of this figure: 26.

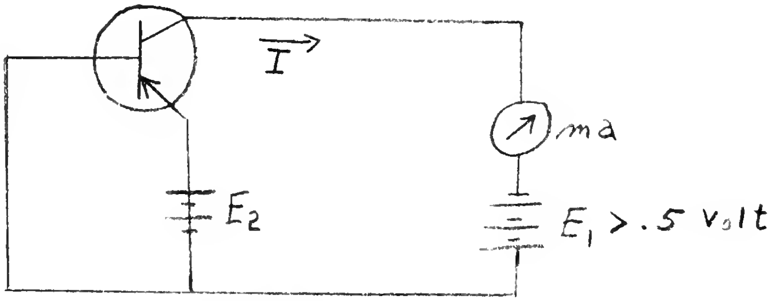

Figure 26

$E_1 > .5$ volt

$$E_2 = \frac{kT}{q} \ln 1000 \text{ or } \frac{kT}{q} \ln 100 = \frac{1}{38.6} \; 6.9 =$$

180 mv at T = 27°C, etc.

$$I_o \doteq \frac{I}{1000}$$

or

$$I_o \doteq \frac{I}{100}$$

I_3 = Quiescent collector current minus I_{co}

I_c = Final value of collector current after signal is applied but not including I_3

$\bar{I}_c$ = Total Collector current

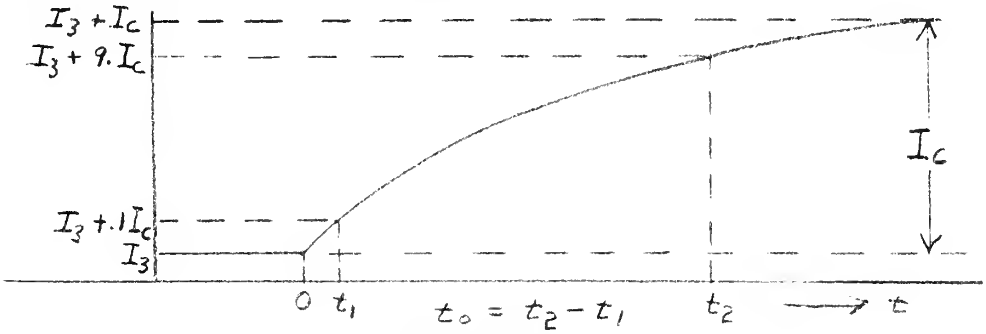

Figure 27

Therefore:

$$V_{b'e} = K \ln \left[\frac{I_3}{I_o} + \frac{I_c}{I_o}\left(1 - e^{-t/T}\right)\right] = K\ln \frac{I_3 + I_c}{I_o} + K \ln\left[1 - \frac{I_c}{I_c + I_3} e^{-t/T}\right]$$

$$I_1 = \frac{\bar{I}_c}{cb}$$

$$C_{b'e} = K_1 \bar{I}_c \qquad K_1 = \Lambda \frac{W_b^2}{2 D_p}$$

$$I_2 = C_{b'e} \frac{d V_{b'e}}{dt} = K K_1 \bar{I}_c \left[\frac{\frac{I_c}{T} e^{-t/T}}{I_c}\right]$$

$$= \frac{K_1 K I_c}{T} e^{-t/T}$$

$$= \frac{I_c}{\alpha_{cb} \omega_{cb} T} e^{-t/T} \qquad \text{Since } K_1 K = \frac{W_b^2}{2 D_p} \doteq \frac{1}{\alpha_{cb} \omega_{cb}}$$

where W_b = base thickness

D_p = diffusion contact for p-type germanium.

The derivation of the relationship is contained in Appendix II.

$$I_b = I_1 + I_2 = \frac{I_3}{\alpha cb} + \frac{I_c\left(1 - e^{-t/T}\right)}{\alpha cb} + \frac{I_c}{\alpha cb\,\omega cb\, T} e^{-t/T}$$

$$E(t) = I_b R_1 + V_{b'e}$$

$$= \frac{I_3 R_1}{\alpha cb} + \frac{I_c R_1}{\alpha cb} - \frac{I_c R_1}{\alpha cb} e^{-t/T} + \frac{I_c R_1}{\alpha cb\,\omega cb\, T} e^{-t/T} +$$

$$\frac{1}{\Lambda} \ln \frac{I_3 + I_c}{I_o} + \frac{1}{\Lambda} \ln \left[1 - \frac{I_c}{I_3 + I_c} e^{-t/T} \right]$$

and $\ln (1 - x) = - x - 1/2\, x^2 - 1/3\, x^3 \ldots\ldots\ldots$

$$-1 < x < 1$$

Therefore:

$$\frac{1}{\Lambda} \ln \left[1 - \frac{I_c}{I_3 + I_c} e^{-t/T} \right] = \frac{-I_c}{\Lambda (I_3 + I_c)} e^{-t/T} - \frac{I_c^2}{2\Lambda (I_3 + I_c)^2} e^{-2t/T} \ldots\ldots$$

Assume $T = \dfrac{1}{n\omega_{cb}}$

Then for a first approximation to a step of voltage for E(t)

$$(n - 1) \frac{I_c R_1}{\alpha_{cb}} e^{-t/T} = \frac{1}{\Lambda} \frac{I_c}{I_3 + I_c} e^{-t/T}$$

$$\frac{n\, I_c\, R_1}{\alpha_{cb}} = \frac{1}{\Lambda} \frac{I_c}{I_3 + I_c} + \frac{I_c\, R_1}{\alpha_{cb}}$$

$$n = \frac{\frac{1}{\Lambda} \alpha_{cb}}{R_1 (I_3 + I_c)} + 1$$

at t_1 $\quad I_c = I_3 + .1\, I_c$

$$I_3 + .1\, I_c = I_3 + I_c (1 - e^{-t/T})$$

$$.9 = e^{-t/T}$$

$$t_1 = T \ln 1.11$$

Similarly:

$$t_2 = T \ln 10$$

$$\therefore t_o = \ln \frac{10}{1.11} = T \ln 9$$

Case 2 - For a better approximation.

Same equivalent circuit but assume:

$$\bar{I}_c = I_3 + I_{c1}(1 - e^{-t/T}) + I_{c2}(1 - e^{-2t/T}) = I_o \, e^{V_{b'e}/K}$$

where $I_c = I_{c1} + I_{c2}$

I_{c2} is an arbitrary current to be derived.

$$V_{b'e} = K \ln \frac{1}{I_o} + K \ln \left[I_3 + I_{c1}(1 - e^{-t/T}) + I_{c2}(1 - e^{-2t/T}) \right]$$

$$I_1 = \frac{\bar{I}_c}{\alpha_{cb}}, \quad C_{b'e} = K_1 \bar{I}_c$$

$$I_2 = C_{b'e} \frac{d V_{b'e}}{dt} = K_1 K \left[I_{c1} \frac{1}{T} e^{-t/T} + I_{c2} \frac{2}{T} e^{-2t/T} \right]$$

$$I_b = I_1 + I_2 = \frac{I_{c3}}{\alpha_{cb}} + \frac{I_{c1}}{\alpha_{cb}}(1 - e^{-t/T}) + \frac{I_{c2}}{\alpha_{cb}}(1 - e^{-2t/T}) +$$

$$+ \frac{1}{\alpha_{cb}\,\omega_{cb}T} \left(I_{c1} e^{-t/T} + 2 I_{c2} \, e^{-2t/T} \right)$$

$$E(t) = \frac{R_1 \; I_3}{\alpha_{cb}} + \frac{R_1 \, (I_{c1} + I_{c2})}{\alpha_{cb}} - \frac{R_1 \, I_{c1}}{\alpha_{cb}} e^{-t/T} + \frac{R_1 \, I_{c1}}{\alpha_{cb}\,\omega_{cb}T} e^{-t/T} - \frac{R_1 \, I_{c2}}{\alpha_{cb}} e^{-/T} +$$

$$+ \frac{2 \, R_1 \, I_{c2}}{\alpha_{cb}\,\omega_{cb} \, T} e^{-2t/T} + \frac{1}{\Lambda} \ln \left(\frac{I_3 + I_c}{I_o} \right) +$$

$$+\frac{1}{\Lambda} \ln \left[1 - \frac{I_{c1}}{I_c + I_3} e^{-t/T} - \frac{I_{c2}}{I_c + I_3} e^{-2t/T} \right]$$

where:

$$\frac{1}{\Lambda} \ln \left[1 - \frac{I_{c1}}{I_c + I_3} e^{-t/T} - \frac{I_{c2}}{I_c + I_3} e^{-2t/T} \right] =$$

$$- \frac{1}{\Lambda} \left[\frac{I_{c1}}{I_c + I_3} e^{-t/T} + \frac{I_{c2}}{I_c + I_3} e^{-2t/T} + \frac{I_{c1}{}^2}{2(I_c + I_3)} e^{-2t/T} + \; \cdots\cdots \right]$$

Assume $T = \dfrac{1}{n\,\omega_{cb}}$

$$(n - 1) \frac{R_1}{\alpha_{cb}} = \frac{1}{\Lambda(I_c + I_3)}$$

$$n = \frac{\alpha_{cb}}{\Lambda R_1 \, (I_c + I_3)} + 1$$

Since $I_c = I_{c1} + I_{c2}$ or, $I_{c1} = I_c - I_{c2}$

$$(2n-1) \quad \frac{R_1 \quad I_{c2}}{\alpha_{cb}} = \frac{I_{c2}}{\Lambda(I_c + I_3)} \quad \frac{(I_c - I_{c2})^2}{2\,\Lambda\,(I_c + I_3)^2}$$

Substituting for n

$$\frac{2\, I_{c2}}{\Lambda(I_c + I_3)} + \frac{R_1\, I_{c2}}{\alpha_{cb}} = \frac{I_{c2}}{\Lambda\,(I_c + I_3)} + \frac{(I_c - I_{c2})^2}{2\Lambda\,(I_c + I_3)^2}$$

$$\frac{2}{\Lambda} \; I_{c2}\,(I_c + I_3) + \frac{R_1}{\alpha_{cb}} \; I_{c2}\,(I_c \quad I_3)^2 - \frac{1}{\Lambda} \; I_{c2}\,(I_c + I_3) +$$

$$- \frac{1}{2\Lambda} (I_c^2 - 2I_c I_{c2} + I_{c2}^2) = 0$$

$$\frac{1}{2\Lambda} I_{c2}^2 - \left[\frac{1}{\Lambda} (I_c + I_3) + \frac{R_1}{\alpha cb} (I_c + I_3)^2 + \frac{1}{\Lambda} I_c \right] I_{c2} + \frac{1}{2\Lambda} I_c^2 = 0$$

$$I_{c2}^2 - \left[4 I_c + 2I_3 + \frac{2\Lambda R_1}{\alpha cb} (I_c \quad I_3)^2 \right] I_{c2} + I_c^2 = 0$$

Therefore:

$$I_{c2} = 2 I_c + I_3 + \frac{\Lambda R_1}{\alpha cb} (I_c + I_3)^2 - \sqrt{\left[2 I_c + I_3 + \frac{R_1}{\alpha cb} (I_c + I_3)^2 \right]^2 - I_c^2}$$

Let $K_4 = 2 I_c + I_3 + \frac{\Lambda R_1}{\alpha cb} (I_c + I_3)^2$

then $I_{c2} = K_4 - (K_4^2 - I_c^2)^{1/2}$

or $I_{c2} = K_4 - K_4 \left[1 - \left(\frac{I_c}{K_4} \right)^2 \right]^{1/2}$

Solving for t_o :

$$\bar{I}_c = I_3 + I_{c1} (1 - e^{-t/T}) + I_{c2} (1 - e^{-2t/T})$$

at $t = t_1$

$$\bar{I}_c = I_3 \quad .1 (I_{c1} + I_{c2})$$

$$.9 (I_{c1} + I_{c2}) = I_{c1} e^{-t1/T} + I_{c2} e^{-2t1/T}$$

$$= I_{c1} e^{-t1/T} (1 \quad \frac{I_{c2}}{I_{c1}} e^{-t1/T})$$

$$t_1 = T \left[\ln \quad 1.11 \quad \frac{I_{c1}}{(I_{c1} + I_{c2})} + \ln \quad \left(1 + \frac{I_{c2}}{I_{c1}} \; e^{-t_1/T}\right) \right]$$

$$= \; T \ln \left[1.11 \quad \frac{I_{c1} + I_{c2} \; e^{-t_1/T}}{I_{c1} + I_{c2}} \right]$$

Assuming $t_1 \doteq .104\ T$ and since $I_c = I_{c1} + I_{c2}$

$$t_1 = T \ln \left[1.11 \quad \frac{I_c - .1 I_{c2}}{I_c} \right]$$

similarly $t_2 = T \ln \left[10 \quad \frac{I_c - .9 I_{c2}}{I_c} \right]$

Hence $t_o = T \ln \left[9 \quad \frac{I_c - .9 I_{c2}}{I_c - .1 I_{c2}} \right] \quad , \quad I_{c2} << I_c$

unless $I_{c2} << I_c$, for really accurate results, the factors .9 and .1 above must be corrected to the more accurate values of t_1 and t_2 and then the most accurate value determined by successive approximations. However, for most values, the above is very close since even if $R_1 = \alpha_{cb}$ which is an exceedingly small value, $I_{c2} = .026$ ma at room temperature and $I_c = 1$ ma, even if $I_3 = 0$ hence even in the extreme, the error is reasonable.

Case 3 - Attempting to include $C_{b'c}$

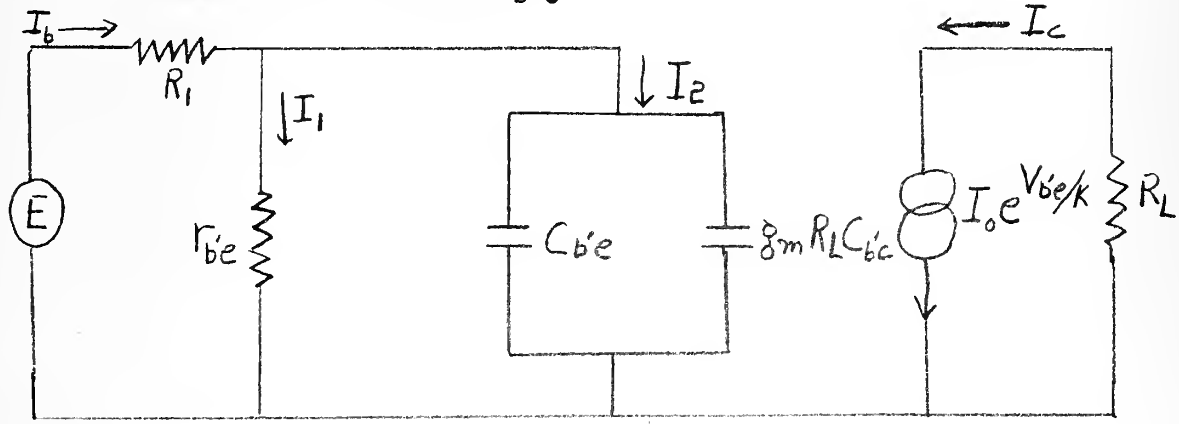

Figure 28

Assume $\bar{I}_c = I_3 + I_c(1 - e^{-t/T}) = I_o e^{V_{b'e}/K}$

$$V_{b'e} = K \ln \frac{1}{I_o} + K \ln \left[I_3 + I_c(1 - e^{-t/T}) \right]$$

$$I_1 = \frac{\bar{I}_c}{\alpha_{cb}}, \quad C_{b'e} = K_1 \bar{I}_c, \quad C_{b'c} = \frac{K_5}{(V_{ce})^{1/2}}$$

$$gm\ R_L C_{b'c} = \frac{gm\ R_L K_5}{(E_{cc} - \bar{I}_c R_L)^{1/2}} = \frac{\bar{I}_c K_6 K_5 R_L}{(E_{cc} - \bar{I}_c R_L)^{1/2}}$$

$$I_2 = \bar{I}_c \left[K_1 + \frac{K_6 K_5 R_L}{(E_{cc} - \bar{I}_c R_L)^{\frac{1}{2}}} \right] \frac{d V_{b'e}}{dt}$$

$$= \left[K_1 + \frac{K_6 K_5 R_L}{(E_{cc} - \bar{I}_c R_L)^{\frac{1}{2}}} \right] \frac{I_c K}{T} e^{-t/T}$$

$$= \frac{I_c}{\alpha_{cb} \omega_{cb} T} e^{-t/T} + \frac{K_5 R_L I_c}{T(E_{cc})^{\frac{1}{2}}} + \frac{K_5 R_L^2 I_c(I_3 + I_c)}{2T(E_{cc})^{3/2}} e^{-t/T} - \frac{K_5 R_L^2 I_c^2}{2T(E_{cc})^{3/2}} e^{-2t/T}$$

since: $(1 - x)^n = 1 - nx + \frac{n(n-1)}{2} x^2 - \frac{n(n-1)(n-2)}{3!} x \ldots$

$$\frac{1}{(E_{cc})^{\frac{1}{2}}} \left(1 - \frac{\bar{I}_c R_L}{E_{cc}}\right)^{-\frac{1}{2}} = 1 + \frac{1}{2} \frac{\bar{I}_c R_L}{E_{cc}} + 3/8 \left(\frac{\bar{I}_c R_L}{E_{cc}}\right)^2 x \ldots$$

$$= \frac{1}{(E_{cc})^{\frac{1}{2}}} + \frac{(I_3 + I_c) R_L}{2(E_{cc})^{3/2}} - \frac{I_c R_L}{2(E_{cc})^{3/2}} e^{-t/T} \ldots\ldots$$

$$K K_1 = \frac{1}{\alpha_{cb} \omega_{cb}} \quad \text{as before}$$

$K K_6 K_5 = K_5$ which is derived later.

$$E(t) = \frac{(I_3 + I_c)\, R_1}{\alpha_{cb}} - \frac{I_c R_1}{\alpha_{cb}} e^{-t/T} + \frac{K_5\, R_L I_c R_1}{T\,(E_{cc})^{\frac{1}{2}}} e^{-t/T} + \frac{K_5 R_1 R_L{}^2 I_c (I_c + I_3)}{2\,T\,(E_{cc})^{3/2}} e^{-t/T} +$$

$$- \frac{K_5\, R_1\, R_L{}^2\, I_c{}^2}{2\,T\,(E_{cc})^{3/2}} e^{-2t/T} + \ldots\ldots\ldots + \frac{I_c\, R_1}{\alpha_{cb}\omega_{cb}T} e^{-t/T} +$$

$$\frac{1}{\Lambda} \ln \frac{I_3 + I_c}{I_o} + \frac{1}{\Lambda} \ln \left(1 - \frac{I_c}{I_3 + I_c} e^{-t/T}\right)$$

where $\frac{1}{\Lambda} \ln \left(1 - \frac{I_c}{I_3 + I_c} e^{-t/T}\right) = - \left[\frac{I_c}{\Lambda(I_c + I_3)} e^{-t/T} + \frac{1}{2}\frac{I_c{}^2}{\Lambda(I_c + I_3)^2} e^{-2t/T} \ldots\right]$

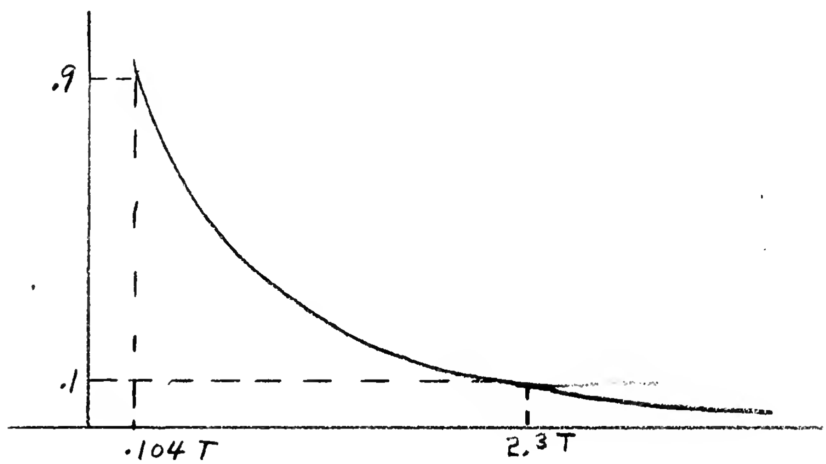

Figure 29

$$\text{time mean} = \frac{1}{2.2T} \int_{.104T}^{2.3T} e^{-t/T}\, dt$$

$$= \frac{1}{2.2T} \Big[- T\, e^{-t/T} \Big]_{.104T}^{2.3T}$$

$$\doteq \frac{1}{2.2}\ (-\ .1 + .9) = \frac{.8}{2.2} = .364$$

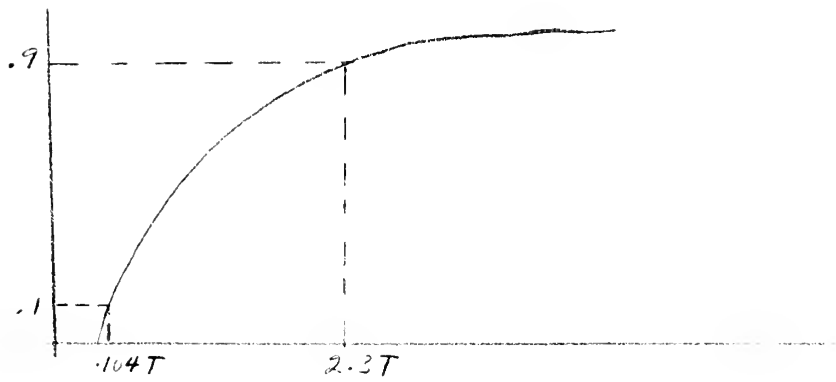

Figure 30

$$\text{time mean} = \frac{1}{2.2T} \int_{.104T}^{2.3T} (\ 1 - e^{-t/T})\ dt$$

$$= \frac{1}{2.2T} \left[\ t + T\ e^{-t/T} \right]_{.104T}^{2.3T}$$

$$= \frac{1}{2.2T} \left[2.3T - .1T + .1T - .9T \right]$$

$$= \frac{1.4}{2.2} = .637$$

Assume $T = 1/n\ \omega cb$

$$n\ I_c\ R_1 \left[\frac{1}{\alpha_{cb}} + \frac{K_5\ R_L \quad cb}{(E_{cc})^{1/2}} \left(1 + \frac{R_L\ (I_3 + I_c)}{2\ E_{cc}} - \frac{.637\ R_L I_c}{2\ E_{cc}} \right) \right] =$$

$$= \frac{I_c}{\Lambda(I_3 + I_c)} + \frac{R_L\ I_c}{\alpha_{cb}}$$

Let the expression in brackets $= \mu$

$$n\, I\, R \mu = \frac{1}{\Lambda (I_c + I_3)} + \frac{R_1}{\alpha_{cb}}$$

$$n = \frac{\dfrac{\alpha_{cb}}{\Lambda\, R_1\, (I_c + I_3)} + 1}{\alpha_{cb} \mu}$$

$$t_o = T \ln 9$$

Another approach to the $C_{b'c}$ problem:

Assume $C_{b'c}$ is about constant

$$I_2 = \bar{I}_c \quad (K_1 + \Lambda\, R_L\, C_{b'c}) \frac{dV_{b'e}}{dt}$$

$$= \left(\frac{1}{\alpha_{cb} \omega_{cb}} + R_L\, C_{b'c} \right) \frac{I_c}{T}\, e^{-t/T}$$

$$= \left(\frac{1 + \alpha_{cb}\, \omega_{cb}\, R_L C_{b'c}}{\alpha_{cb}\, \omega_{cb}} \right) \frac{I_c}{T}\, e^{-t/T}$$

$$I_b = \frac{I_{c3}}{\alpha_{cb}} + \frac{I_c\, (1 - e^{-t/T})}{\alpha_{cb}} + \frac{I_c\, (1 + \alpha_{cb}\, \omega_{cb}\, R_L\, C_{b'c})}{\alpha_{cb}\, \omega_{cb}\, T}\, e^{-t/T}$$

E(t) same as before without $C_{b'c}$ term except term:

$$\frac{I_c}{\alpha_{cb}\, \omega_{cb} T}\, e^{-t/T} \quad \text{is now} \quad \frac{I_c(1 + \alpha_{cb}\, \omega_{cb}\, R_L\, C_{b'c})}{\alpha_{cb}\, \omega_{cb}\, T}\, e^{-t/T}$$

Assume $T = 1/n\ \omega_{cb}$ as before

$$- \frac{I_c R_1}{\alpha_{cb}} + \frac{n\, I_c\, (1 + \alpha_{cb}\, \omega_{cb}\, R_L\, C_{b'c}) R_1}{\alpha_{cb}} - \frac{1}{\Lambda}\left[\frac{I_e}{I_3 + I_c}\right] = 0$$

$$n\, (1 + \alpha_{cb}\, \omega_{cb}\, R_L\, C_{b'c}) = \frac{\alpha_{cb}}{\Lambda R_1\, (I_3 + I_c)} + 1$$

$$n = \frac{\dfrac{\alpha_{cb}}{\Lambda R_1 (I_3 + I_c)} + 1}{1 + \alpha_{cb}\, \omega_{cb}\, R_L\, C_{b'c}}$$

Using the simple approach for $C_{b'c}$ for Case 2:

$$I_2 = \frac{1 + \alpha_{cb}\, \omega_{cb}\, R_L\, C_{b'c}}{\alpha_{cb}\, \omega_{cb}}\left[\frac{I_{c1}}{T}\, e^{-t/T} + \frac{2I_{c2}}{T}\, e^{-2t/T}\right]$$

I_b same as before except I_2 term changed.

$E(t)$ same as before except changes terms from I_2

Therefore: $n = \dfrac{\dfrac{\alpha_{cb}}{\Lambda R_1 (I_c + I_3)} + 1}{1 + \alpha_{cb}\, \omega_{cb}\, R_L C_{b'c}}$

$$\frac{2n\, (1 + \alpha_{cb}\, \omega_{cb}\, R_L\, C_{b'c})\, R_1}{\alpha_{cb}\, \omega_{cb}\, T} - \frac{R_1}{\alpha_{cb}} = \frac{I_{c2}}{\Lambda (I_c + I_3)} + \frac{(I_c - I_{c2})^2}{2\, \Lambda\, (I_c + I_3)^2}$$

putting in the value of n cancels out constant so value of I_2 is unaffected by this term.

Solving for K_5:

$$\frac{K_5}{(V_{ce})^{\frac{1}{2}}} \doteq \frac{K_e \epsilon_o A_c}{\left(\frac{2K_e \epsilon_o \mu_n V_{ce}}{\sigma_b}\right)^{\frac{1}{2}}}$$

Numbers for units being used:

K_e = relative permativity = 16 for germanium

ϵ_o = permativity of free space = $1/36\pi \times 10^{-9}$ farads/meter

A_c = junction area in meters^2 = $\pi \times 6.42 \times 10^{-8}$ meters^2

μ_n = mobility of electrons in n-type germanium in meters^2/volt sec. $\doteq$ .38 for σ_n = 100 mhos/meter

σ_b = conductivity of n-type germanium in mhos/meter = 100 mhos/meter

$$\frac{K_5}{(V_{ce})^{\frac{1}{2}}} = \frac{2.85 \times 10^{-17}}{(.107 \times 10^{-11} V_{ce})^{\frac{1}{2}}} = \frac{27.5 \times 10^{-12}}{(V_{ce})^{\frac{1}{2}}}$$

therefore $K_5 = 27.5 \times 10^{-12}$

Another method for an approximate value practically is:

$$(V_{ceo})^{\frac{1}{2}} C_c = K_5$$

where V_{ceo} here is that value where C_c is measured.

FALL TIME

Case 1: Same equivalent circuit without $C_{b'c}$

Assume: $\bar{I}_c = I_4 + I_c\, e^{-t/T} = I_o\, e^{V_{b'e}/K}$

where the symbols are as in the rise time derivations except I_4 here corresponds to I_3 in the rise time expressions.

$$V_{b'e} = K \ln \frac{1}{\bar{I}_o} + K \ln (I_4 + I_c\, e^{-t/T})$$

$$I_1 = \frac{\bar{I}_c}{cb} \quad , \quad C_{b'e} = K_1\, \bar{I}_c$$

$$I_2 = C_{b'e} \frac{d\, V_{b'e}}{dt} = - \frac{I_c}{\alpha_{cb}\, \omega_{cb}\, T} e^{-t/T}$$

$$I_b = \frac{I_4}{\alpha_{cb}} + \frac{I_c}{\alpha_{cb}} e^{-t/T} - \frac{I_c}{\alpha_{cb}\, \omega_{cb}\, T} e^{-t/T}$$

$$E(t) = \frac{I_4\, R_1}{\alpha_{cb}} + \frac{I_c\, R_1}{\alpha_{cb}} e^{-t/T} - \frac{I_c\, R_1}{\alpha_{cb}\, \omega_{cb}\, T} e^{-t/T} + \frac{1}{\Lambda} \ln \left(\frac{I_4}{I_o}\right) +$$

$$+ \frac{1}{\Lambda} \ln \left(1 + \frac{I_c}{I_4} e^{-t/T}\right) + E_o$$

Where E_o = initial value of step

$$\ln (1+x) = 2 \left(\frac{x-1}{x+1}\right) + \frac{1}{3} \left(\frac{x-1}{x+1}\right)^3 + \frac{1}{5} \left(\frac{x-1}{x+1}\right)^5 \; . \; . \; . \; . \; .$$

where $x > 0$

$$\frac{1}{\Lambda} \ln \left(1 + \frac{I_c}{I_4} e^{-t/T}\right) \doteq \frac{2 I_c e^{-t/T}}{\Lambda (2 I_4 + I_c e^{-t/T})}$$

Let $T = 1/n\, \omega_{cb}$

$$(1-n) \frac{R_1}{\alpha_{cb}} = - \frac{2}{\Lambda (2I_4 + I_c e^{-t/T})}$$

$$n = 1 + \frac{2 \alpha_{cb}}{\Lambda R_1 (2 I_4 + I_c e^{-t/T})}$$

using time means as before.

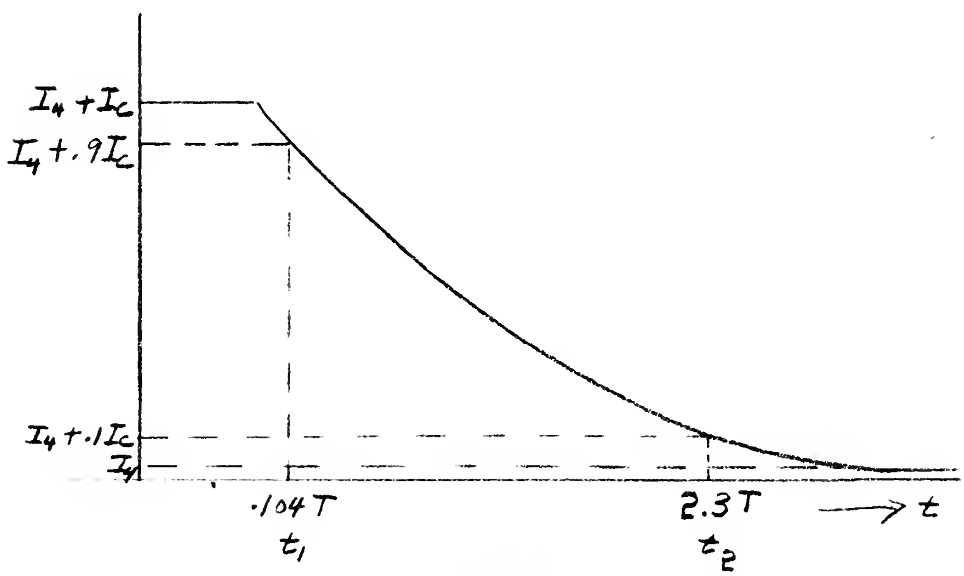

Figure 31

$t_o = t_2 - t_1$

$t_1 = T_1 \ln 1.11$

$t_2 = T_2 \ln 10$

$$n_1 = 1 + \frac{2\,\alpha_{cb}}{\Lambda R_1 \,(2\, I_4 + .95\, I_c)}$$

$$n_2 = 1 + \frac{2\,\alpha_{cb}}{\Lambda R_1 \,(2\, I_4 + .364\, I_c)}$$

$$T_1 = \frac{1}{n_1\, \omega_{cb}} \,, \quad T_2 = \frac{1}{n_2\, \omega_{cb}}$$

$$t_o = T_2 \ln 10 - T_1 \ln 1.11$$

In practice, .1 works better here than .364 due to the other terms in the series that were disregarded.

To include $C_{b'c}$ as before $\quad \overline{I}_c = I_4 + I_c\, e^{-t/T}$

$$I_2 = - \frac{I_c}{\alpha\, cb\, \omega cb\, T} e^{-t/T} + \frac{K_5\, R_L\, I_c}{T\,(E_{cc})^{\frac{1}{2}}} e^{-t/T} + \frac{K_5\, R_L{}^2\, I_c\, I_4}{2\,T\,(E_{cc})^{3/2}} +$$

$$+ \frac{K_5\, R_L{}^2\, I_c{}^2}{2\,T\,(E_{cc})^{3/2}} e^{-2t/T} \quad \ldots\ldots\ldots\ldots$$

$$I_b = \frac{I_4}{\alpha_{cb}} + \frac{I_c}{\alpha_{cb}} e^{-t/T} + I_2$$

$$E(t) = \frac{I_4\, R_1}{\alpha\, cb} + \frac{I_c\, R_1}{\alpha cb} e^{-t/T} - \frac{I_c\, R_1}{\alpha cb\, \omega cb\, T} e^{-t/T} + \frac{K_5\, R_L\, R_1\, I_c}{T\,(E_{cc})^{\frac{1}{2}}} e^{-t/T} +$$

$$+ \frac{K_5 R_1 R_L^2 I_c I_4}{2T(E_{cc})^{3/2}} e^{-t/T} + \frac{K_5 R_1 R_L^2 I_c^2}{2T(E_{cc})^{3/2}} e^{-2t/T} \cdots\cdots +$$

$$+ \frac{1}{\Lambda} \ln\left(\frac{I_4}{I_o}\right) + \frac{2 I_c e^{-t/T}}{\Lambda(2I_4 + I_c e^{-t/T})} \cdots\cdots\cdots + E_o$$

let $T_1 = 1/n_1 \omega_{cb}$

$T_2 = 1/n_2 \omega_{cb}$

$$n = \frac{1 + \dfrac{2\alpha_{cb}}{\Lambda R_1 (2I_4 + I_c e^{-t/T})}}{\alpha_{cb}\mu}$$

$$n_1 = \frac{1 + \dfrac{2\alpha_{cb}}{\Lambda R_1 (2I_4 + .95 I_c)}}{\alpha_{cb}\mu}$$

$$n_2 = \frac{1 + \dfrac{2\alpha_{cb}}{\Lambda R_1 (2I_4 + .364 I_c)}}{\alpha_{cb}\mu}$$

where $\mu = \dfrac{1}{\alpha_{cb}} + \dfrac{K_5 R_L \omega_{cb}}{(E_{cc})^{\frac{1}{2}}} \left[1 + \dfrac{R_L I_4}{2 E_{cc}} + \dfrac{R_L I_c}{2 E_{cc}} \right]$

.364 was not used as the coefficient of the term $\dfrac{R_L I_c}{2 E_{cc}}$ since many terms that follow add to this number and eventually they would

increase it to one in the limit and hence one is a better approximation.

$$t_o = T_2 \ln 10 - T_1 \ln 1.11$$

To consider the $C_{b'c}$ as it was done before in the simpler case yields:

$$n_1 = \frac{1 + \dfrac{2\alpha_{cb}}{\Lambda R_1 (2I_4 + .95\ I_c)}}{1 + \alpha_{cb}\,\omega_{cb}\ R_L\ C_{b'c}}$$

$$n_2 = \frac{1 + \dfrac{2\alpha_{cb}}{\Lambda R_1 (2I_4 + .364\ I_c)}}{1 + \alpha_{cb}\,\omega_{cb}\ R_L\ C_{b'c}}$$

etc.

In practice this formula works better in all cases tested if .1 is used instead of .364.

Thesis
B138

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Bailey
The operation of alloy junction transistors as large signal pulse amplifiers in the active region.